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SOME WELFARE EFFECTS OF INFLATION

Roland Benabou

No. 542 November 1989

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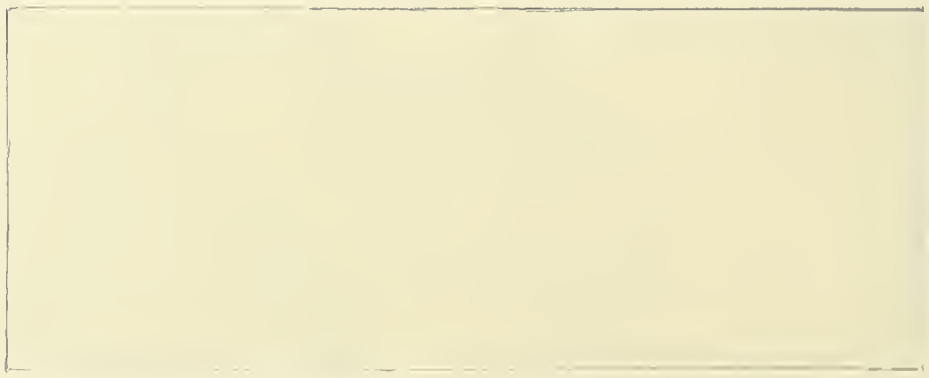

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ROLAND BENABOU

MIT

September 1989

Revised November 1989

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Some Welfare Effects of Inflation

Roland Benabou

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ABSTRACT

This paper examines the effects of inflation on the efficiency of the price system, output and welfare, through a model of monopolistic competition with consumer search and (S,s) price setting. Its results have relevance for both search theory and macroeconomics. In particular, it provides a foundation for the conventional wisdom linking higher inflation, through increased price dispersion, to more resources spent on search. It also brings to light some efficiency-improving effects, as well as others whose impact depends on preferences and market structure. The paper's theoretical analysis and numerical simulations thus deliver a message of caution against simple arguments about the "distortions" caused by inflation.

INTRODUCTION

"A persistent theme in the inflation literature is that inflation interferes with the efficient allocation of the price system", as noted by Fischer [1984]. One of the channels through which this distortion may operate is often thought to be an increase in relative price variability and dispersion, leading to both inefficient production decisions and more resources being spent on search.

Indeed, there is substantial empirical evidence that higher rates of inflation are associated with increased price variability, both across goods (e.g. Fischer [1981] and the references therein, or Taylor [1981]) and within markets for homogeneous goods (Domberger [1987], Dantziger [1987b]). Several theories, surveyed in Fischer [1981], can potentially account for this correlation. There is, however, no model (nor any empirical study) of the hypothesized link between inflation and the resource cost of search.

The aim of this paper is to assess the effects of inflation on price dispersion, search costs, the efficiency of the price system, output and ultimately social welfare, through a rigorous model of monopolistically competititive price setting and consumer search. The model is essentially micro-theoretic, but individual strategies can be aggregated (in spite of the absence of a representative agent), generating macroeconomic conclusions.

Inflation is understood here in the classical sense, as a uniform and anticipated growth rate of aggregate prices. The relevant question is thus superneutrality, and whether 20% annual inflation is worse or better than 10%, and 10% worse or better than 0%. If higher average inflation also means more volatile inflation, there will be other effects in addition to those identified here. Benabou and Gertner [1988] look at the effects on prices and welfare of unanticipated inflation, i.e. focus on the variance rather than trend of

aggregate prices, as in Lucas [1973], Cuckierman [1979] [1983] or Hercowitz [1981]. Interestingly, once agents' ability to acquire information through search is recognized, similar conclusions emerge from the two, otherwise dissimilar, models.

Benabou [1988] showed that in a search market where price adjustment is costly, inflation can increase price dispersion, thereby intensifying competition, reducing real prices and increasing welfare (entry equalizes profits to zero). Diamond [1988], using different assumptions about search and price adjustment technologies, obtains similar effects for moderate inflation rates (according to simulations); but as inflation increases further, a worsening of the "thin-market" externality prevails and welfare declines. Both models, however, assume that buyers are identical, so that in equilibrium they never search voluntarily, and have unit demand, so that prices' only allocative role is through entry. They are thus of limited use in analysing the resource cost of search and the impact of inflation on the efficiency of the price system.

An alternative to search as a basis for imperfect competition is product differentiation. Naish [1986] examines the effects of inflation on a monopolists' output. Dantziger [1988] extends the analysis to an economy consisting of a large, fixed number of monopolists facing a competitive sector and finds that a little inflation improves welfare. Konieczny [1987] allows for competition between firms selling differentiated products, but still not for entry. He also incorporates labor productivity and the transactions services of money as general equilibrium channels through which inflation operates, and finds it to be welfare-reducing.[1] Both his and Dantziger's results, however, are in good part predicated on the specific utility functions which they use.

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[1] He does not, however, prove that an equilibrium exist.

In fact, as this paper will show, consumer preferences and market structure, including the entry and exit of firms, both matter crucially for output and welfare conclusions.

To address the issues raised above, this paper generalizes the monopolistically competitive model of Benabou [1988] in two directions:

(i) by incorporating heterogeneity among buyers, it accounts for active search in equilibrium, and allows its resource cost to be evaluated.

(ii) by endowing buyers with very general preferences, it takes into account in a robust manner the full allocative role played by prices in the determination of output and welfare. In particular, the dependence of equilibrium transactions prices on the rate of inflation generates a long-run Phillips curve, which does not rely on any misperceptions.

These generalizations make an equilibrium a fairly complex object. Firms follow staggered (S,s) strategies while buyers search sequentially, and each agent's strategy is a best reponse to those of all the others. Typically, buyers with a low search cost seek and find firms which have not revised their nominal price recently, while those with higher search cost search less and buy more dearly. Correspondingly, each firm's sales increase over the time during which its nominal price remains fixed, and fall after each adjustment. Given the added complexity, one can still prove the existence of an equilibrium and analyze the different components of welfare, but for comparative statics one must turn to simulations.

The existence result is interesting in itself, because one of its underpinnings is the way in which the demand functions (derived from search) of heterogeneous individuals aggregate so as to induce a quasi-concave profit function. It thus parallels, in a context of search and dynamic price-setting, the results established by Caplin and Nalebuff [1988] for a wide class of (static) models of product differentiation.

As to the simulations, they confirm that the model provides a micro-theoretical basis for the hypothesized link between higher inflation, increased price dispersion and the increased resources expended on search. They also shed light on some of the empirical puzzles relative to price adjustment documented by Cechetti [1986] and Kashyap [1986]. But most importantly, they serve as a warning that informal discussions of the "distortions" caused by inflation may be misleading when dealing with an imperfectly competitive market. Indeed, inflation is shown to have several real effects in addition to increasing search costs. Some, like the reduction in the number of firms in the market, are welfare-improving; others, such as its impact on the distribution of real transaction prices, depend crucially on consumer preferences and market structure.

Section 1 presents the model. Section 2 analyzes optimal search and price strategies, as well as entry. The existence of an equilibrium is established in Section 3, as well as analytic results about inflation's effects in a simple case. Finally, in Section 4, the different components of welfare are analyzed, and their variation with inflation is examined through simulations.

1 - THE MODEL

1.1 A monopolistically competitive search market. A continuum of identical firms produce a homogeneous good, using labor as their input. They are infinitely-lived with discount rate ρ. During each unit of time, production requires a fixed cost of $h \geq 0$ and a marginal cost $c > 0$ per unit of output. Both are real costs, expressed in units of labor. Firms can change their price at any time, but doing so entails a real fixed cost $\beta > 0$: goods must be relabelled, new price lists and catalogues drawn up and sent, etc; β can also be viewed as a proxy for any adverse reactions of customers to a price increase (say, through reputation effects) not captured by the model. Entry determines the number (density) $\nu \in (0,+\infty)$ of firms operating in equilibrium.

During each interval of time of length dt, a flow 1.dt of buyers enter the market to make a single purchase of the good (for instance a durable). They derive from buying it at a real price p a net surplus or indirect utility V(p).

Assumption 1: Consumer surplus V(p) is decreasing, convex and C_2 inside its support [0,M], $c < M \leq +\infty$. Moreover, $z(p) \equiv -V'(p)$ has a non-decreasing elasticity $\alpha_z(p) \equiv -pz'(p)/z(p)$ on (0,M); finally, if $M = +\infty$, then $\alpha_z(+\infty) > 1$.

Assumption 1 is quite weak, and satisfied by all usual demand functions (and associated surplus $V(p) = \int_p^{\infty} z(u)du$: reservation price, linear, exponential, isoelastic, etc.[2] In a monopoly situation, buyers' demand function would be z(p), which will be called the monopoly demand. Under Assumption 1, the corresponding profit function has the following well-known property:

Proposition 1: The monopoly profit function $\pi(p) \equiv (p-c)z(p)$ is strictly quasiconcave on its support, with a maximum at a finite $p^m \in (c,M]$.

We consider, however, a market where buyers (say, consumers) face many sellers, whose individual prices they do not know, but among which they can search. Firms' equilibrium demand and profit functions will then be endogenous, and not equal to z(p) and $\pi(p)$; Proposition 1 will nonetheless prove useful.

It will be assumed that search is instantaneous and that buyers cannot postpone their consumption; thus within the interval dt, all 1.dt of them enter, search, buy, and exit the market, to be replaced by a similar generation of "instantaneous" consumers an instant later.[3] A first price

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[2] When $M < +\infty$, z(p) may also be discontinuous at M, or have an elasticity smaller than one on [0, M].

[3] Search is thus very fast with respect to real price changes. This is the converse of the assumption in Cassella and Feinstein [1987], where inflation alters real prices faster than bargaining offers can be made.

quotation is received for free but each subsequent search requires the expenditure of a fixed amount γ of labor.[4] Consumers maximize the expectation of $V(p)-K(p)\gamma$, where p is the (random) real price at which they end up buying, after K(p) costly searches. They differ only by these search costs, and for simplicity we assume:

Assumption 2: Consumers' search costs are distributed with uniform density $q(\sigma) = \frac{1}{\bar{\gamma} - \underline{\gamma}}$ on $[\underline{\gamma},\bar{\gamma}]$, $0 \leq \underline{\gamma} < \bar{\gamma}$.

1.2 Inflation and (S,s) policies. In a steady, perfectly anticipated inflationary regime, all aggregate prices and costs (in particular the nominal cost of labor, with respect to which real prices are defined) grow at a constant rate of $g > 0$. Under quite general conditions, a firm's optimal price policy in such an environment is an (S,s) rule: it adjusts its nominal price so as to achieve a real value of S every time this real value has been eroded down to $s < S$ (Sheshinski and Weiss [1977], Caplin and Shesinski [1987]).[5]

Our focus will therefore be on (symmetric) equilibria where, given that other firms' strategies are described by a common (S,s) rule, and given buyers' search strategies which constitute their best responses to these prices, it is optimal for any firm, at any point in time and whatever its previous price, to adopt the same (S,s) strategy as its competitors.

---

[4] This standard assumption ensures that no consumer is kept out of the market because the surplus he can expect from optimal search is smaller than the cost of the first sampling. As usual, it is required to ensure that an equilibrium always exists.

[5] The full specification of an (S,s) strategy involves an additional trigger point $\hat{s} > S$, such that if $p > \hat{s}$, the firm adjusts down to S, but if $S < p < \hat{s}$ it lets inflation gradually do the job. For simplicity, and in accordance with the literature, we refer to the strategy as an (S,s) rule, but it should be clear that it implicitly includes an $\hat{s}$, which is relevant to describe the firm's out-of-equilibrium behavior.

In addition, macroeconomic consistency requires that the equilibrium distribution of firms' real prices be invariant over time; otherwise, no aggregate index of firms' nominal prices will grow smoothly at the rate g assumed for the rest of the (macro) economy. Moreover, a time-varying (non degenerate) price distribution would result in non-stationary search rules and demand, and stationary (S,s) price strategies would fail to be optimal.[6] When firms follow identical (S,s) strategies, there exists in fact a unique invariant distribution of real prices, namely the log-uniform distribution on [s,S] (Caplin and Spulber [1987]). With a constant rate of inflation, this corresponds to price adjustments which are uniformly staggered over time (Rotemberg [1983]). Finally, a variety of realistic generalizations of the model imply convergence to the steady-state distribution from any initial conditions.[7] Macroeconomic consistency, optimality and long-run stability thus all require that in equilibrium firms' real prices be distributed on [s,S] according to: [8]

$$(1) \qquad dF(p) = \frac{dp}{p\ \mathrm{Ln}(S/s)}\ , \quad \text{for all } p \text{ in } [s,\ S].$$

1.3 Entry and Equilibrium. The price distribution (1) resulting from firms' (S,s) strategies induces optimal search rules for buyers; in equilibrium, these must aggregate to a demand curve which validates the original strategies

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[6] For any distribution of search costs with low enough γ.

[7] Convergence will occur if firms use slightly different (S,s) bounds, (Caplin and Spulber [1987]), if they are subject to cummulative idiosyncratic shocks (Tsiddon [1987], Caballero and Engle [1989]), or if they randomize their price adjustements to deter storage by speculators (Benabou [1989])

[8] We thus focus on certain types of equilibria, namely staggered symmetric (S,s) equilibria, but do not impose any restriction on strategies. These equilibria are subgame perfect, i.e. such that no firm ever wants to change the nature or timing of its price decisions, as long as no positive measure of firms or buyers have deviated. Other equilibria, e.g. synchronized (S,s), exist but do not satisfy macroeconomic consistency and are not stable to idiosyncratic shocks.

as optimal. Finally, firms enter or exit the market until a marginal entrant's present discounted value of real operating profits (net of adjustment costs) just covers the fixed costs h/ρ. Note that an entrant must also pay β to set its first price (to S optimally), so it is in the same position as a firm at s, about to start a new cycle. Summarizing, we have:

Definition: A stationary, symmetric equilibrium with entry is a triplet (S,s,ν) and a sequential search strategy for each buyer such that:

(i) There are ν firms in the market, whose price strategies consist of a common (S,s) rule and generate an invariant, log-uniform distribution of real prices.

(ii) These price strategies and buyers' search strategies form a subgame perfect equilibrium.

(iii) Firms earn zero net discounted profits over each price cycle.

2 - SEARCH, DEMAND, AND PRICING IN EQUILIBRIUM

2.1 Search. The optimal search rule for a buyer with search cost γ is characterized by a real stopping price r which equates the marginal cost and expected marginal benefit of search:

$$(2) \qquad \int_s^r [V(p)-V(r)]dF(p) = \gamma$$

assuming for now that this equation has a solution.[9] Integrating by parts:

$$(3) \qquad \Gamma(r) \equiv \int_s^r z(p)F(p)dp = \gamma$$

The return $\Gamma(r)$ to searching rather than accepting a price r is increasing and C_1 in r on [s, M); hence Γ has an inverse $R \equiv \Gamma^{-1}$: $[0, \Gamma(M)) \longrightarrow [s, M)$ with

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[9] Since utility is linear in search expenditures and there is no limit to the number of searches a consumer can conduct (we shall return to this question later), optimal search is the same with and without recall.

the same properties. For $\gamma < \Gamma(M)$, (3) has a unique solution $r = R(\gamma)$, which is the optimal reservation price of a consumer with search cost γ. The value of being in the market to such a consumer is then $V(r) + \gamma$, due to the first free search. For $\gamma > \Gamma(M)$, any offer below M is preferable to search, so let $R(\gamma) \equiv M$. The value of being in the market is then the average surplus:

$$(4) \qquad \bar{V}(S,s) \equiv \int_s^S V(u)dF(u) = \int_s^M V(u)dF(u) = \int_s^M z(u)F(u)du = \Gamma(M)$$

because $V(M) = 0$ and in an equilibrium $S \leq M$: no firm ever charges more than M (this intuitive property will be proved below). Thus, a buyer with arbitrary search cost γ rejects offers above his reservation price $r = R(\gamma)$:

$$(5) \qquad R(\gamma) = \sup\{\, r \in (s,M) \mid \Gamma(r) = \int_s^r z(p)F(p)dp \leq \gamma \,\}$$

which embodies both his preferences and search prospects. When faced with an offer $p \leq R(\gamma)$, he accepts it and buys $z(p)$ units. The value of this optimal strategy is $V(R(\gamma)) + \Gamma(R(\gamma))$.[10] The highest and lowest reservation prices in the population $R(\bar{\gamma})$ and $R(\underline{\gamma})$ will be denoted as $\bar{r}$ and $\underline{r}$.

2.2 Demand and profits. The demand curve faced by each firm will now be derived by aggregating the search rules of individual buyers . There are $q(\gamma)$ buyers with search cost γ. They search at random until they find one of the $\nu.F(R(\gamma))$ firms charging a price $p \leq R(\gamma)$, i.e such that $\Gamma(p) \leq \gamma$; each of these firms will therefore eventually retain $q(\gamma)/(\nu.F(R(\gamma)))$ such buyers. Since the probability of discovering such a firm in each round of search is $F(R(\gamma))$, this total will consist of $q(\gamma)/\nu$ buyers on their first search, $q(\gamma)(1 - F(R(\gamma)))/\nu$ on their second search, $q(\gamma)(1 - F(R(\gamma))^{k-1})/\nu$ on their

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[10] If the first search was costly, the consumers with $\gamma > \Gamma(M)$ would stay out of the market. One might therefore be tempted to restrict attention to the case where $\Gamma(M) > \bar{\gamma}$, which also simplifies most equations and proofs. Such an equilibrium, however, does not always exist.

k-th search, etc. The number of consumers who buy from any firm charging a real price p is therefore:

$$(6) \qquad n(p) = \frac{1}{\nu}\left[\int_{\Gamma(p)}^{\infty} \frac{q(\gamma)}{F(R(\gamma))}\, d\gamma\right] \quad \text{for all } p \geq \underline{r};$$

For non-searchers, $R(\gamma) = M \geq S$ so $F(R(\gamma)) = 1$. For searchers, $\gamma = \Gamma(r)$; since Γ is increasing on $(\underline{r}, \bar{r})$, with $\Gamma'(r) = z(r)F(r)$, this change of variables leads to a formulation in terms of reservation prices:

$$(7) \qquad n(p) = \frac{1}{\nu}\left[\int_{p}^{\infty} q(\Gamma(r))z(r)dr + \int_{\Gamma(M)}^{\infty} q(\gamma)d\gamma\right] \quad \text{for } p \geq \underline{r},$$

and $n(p) = n(\underline{r})$ for $p \leq \underline{r}$. Since each buyer purchases $z(p)$ units, the equilibrium demand curve is simply:

$$(8) \qquad D(p) = z(p).\min(n(p), n(\underline{r})).$$

For $p > \bar{r}$, $D(p) = 0$; for $p \leq \bar{r}$, given the assumed uniform distribution of search costs, (8) takes the form:

$$(9) \qquad D(p) = \frac{1}{\bar{\gamma}-\underline{\gamma}} \frac{z(p)}{\nu}\left[\min(V(p),V(\underline{r})) - V(\bar{r}) + \Omega\right]$$

where $\Omega \equiv \min\{\bar{\gamma}-\underline{\gamma}, \max(0,\bar{\gamma}-\Gamma(M))\}$ and $\Omega/(\bar{\gamma}-\underline{\gamma})$ represents the mass of buyers with reservation price equal to M; thus $V(\bar{r}).\Omega = 0$.

Because of search, D(p) has higher elasticity than the original monopoly demand curve z(p) at all prices $p \geq \underline{r}$. Since demand is zero above $\bar{r}$, as all buyers prefer to search, intuition suggests that it is <u>never optimal</u> for a firm to adjust to a price higher than $\bar{r}$, so $S \leq \bar{r} \leq M$. This result, which is formally proved in the next section, leads to a simplification of (9):[11]

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[11] If $\bar{V}(S,s) = \Gamma(M) \geq \bar{\gamma}$, $\Omega = 0$ and, since $\bar{r} \in [S,M]$: $\bar{\gamma} = \Gamma(\bar{r})$ $= \Gamma(M) - \int_{\bar{r}}^{M} z(p)F(p)dp = \Gamma(M) - V(\bar{r})$; thus $-V(\bar{r}) = \bar{\gamma} - \Gamma(M) < \bar{\gamma} - \underline{\gamma}$. If $\Gamma(M) < \bar{\gamma}$, $V(\bar{r}) = V(M) = 0$, while $\Omega = \min\{\bar{\gamma}-\underline{\gamma}, \bar{\gamma}-\Gamma(M)\}$.

$$(10) \qquad D(p) = \frac{1}{\bar{\gamma}-\underline{\gamma}} \frac{z(p)}{\nu}\Big[\min(V(p),V(\underline{r})) + \bar{\gamma} - \max(\underline{\gamma}, \bar{V}(S,s))\Big]$$

This equilibrium demand function is non increasing and continuous, except possibly at $\bar{r}$ when $\bar{r} = M$.[12] Although its elasticity is not necessarily increasing, it has the following fundamental property:

Proposition 2: If $\underline{r} \leq p^m$, the equilibrium profit function $\Pi(p) \equiv (p-c)D(p)$ is strictly quasiconcave on its support $[0,\bar{r}]$, with a maximum at some $p^* \leq p^m$.

Proof: On $(0,\underline{r}]$, $\Pi(p)$ is proportional to monopoly profits $\pi(p)$, hence increasing when $\underline{r} \leq p^m$. The proof that $\Pi(p)$ is strictly quasiconcave on $[\underline{r},+\infty)$ relies on Assumption 1 and is given in appendix.

As in Caplin and Nalebuff [1988], the aggregation of heterogeneous individual demand functions results in a (strictly) quasi-concave profit function.[13] This property will play a doubly crucial role in the existence of an equilibrium, ensuring both that a firms' optimal strategy is an (S',s') rule, and the continuity of this best response in (S,s) space.

2.3 Price strategies. Given a steady inflationary environment and a fixed adjustment cost, the strict quasiconcavity of a firms' profit function is necessary and sufficient (Sheshinski and Weiss [1977], Zinde-Walsh [1986], Caplin and Sheshinski [1987]) to ensure that its optimal price strategy is

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[12] If $z(M^-) > 0$ and $\bar{V}(S,s) = \Gamma(M) < \bar{\gamma}$.

[13] In spite of the definite similarity in the results and the assumptions underlying them, there appears to be no direct mapping of this search problem into a product differentiation model which would allow their powerful aggregation theorem to be used. A direct proof is thus required.

a unique, stationary (S',s') rule.[14] Let T' = Ln(S'/s')/g denote the periodicity of adjustments; intertemporal real operating profits are:

$$(11) \qquad W(S',s') = \frac{\int_0^{T'} \Pi(S'e^{-gt})e^{-\rho t}dt - \beta}{1 - e^{-\rho T'}}$$

This which can also be written (let $u = S'e^{-gT'}$) as:

$$(12) \qquad W(S',s') = \frac{\int_{s'}^{S'} \Pi(u)u^{-1+\rho/g}\, du - \beta g S'^{\rho/g}}{g(S'^{\rho/g} - s'^{\rho/g})}$$

which the firm maximizes over (S',s'). If there exists an interior optimum, it solves the first order conditions $\partial W(S',s')/\partial S' = 0$, $\partial W(S',s')/\partial s' = 0$, or:

$$(13) \qquad \Pi(s') = \rho W(S',s')$$

$$(14) \qquad \Pi(S') - \Pi(s') = \rho\beta$$

Conversely, if (13)-(14) have a solution, it is unique and is an interior optimum (Sheshinski and Weiss [1977]). Condition (13) equates the benefit and opportunity cost of delaying adjustment by dt when s' is reached: additional profits of $\Pi(s').dt$ are earned, but the present value of profits W is deferred by dt. Integrating (12) by parts, condition (14) can be rewritten as equating marginal discounted profits over the price cycle $\int_s^S \Pi'(u)u^{\rho/g}du$ to zero.

In fact the optimum need not be interior, but it always satisfies $S' < \bar{r}$, as assumed all along for the equilibrium S. Indeed, when $\bar{r} = M < +\infty$, D(p)

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[14] There are two additional wrinkles. First, profits must tend to zero as the price tends to $+\infty$; this holds under Assumption 1. Secondly, profits along this (S's') path must remain positive, or else the firm will not always meet demand, as was implicitly assumed, and may even shut down. For a monopolist, a strategy with these properties (even one leading to a non-negative value of the firm) does not always exist -say if price adjustment costs are two large. Here, however, the entry or exit of firms will ensure that it always does.

and $\Pi(p)$ may be discontinuous at $\tilde{r}$ (see (10)), making W not differentiable in S' at that point. The most general first-order condition for S' is therefore: $\partial W(S'^{-},s)/\partial S' \leq 0 \leq \partial W(S'^{+},s)/\partial S'$, with equality unless $S' = \tilde{r}$. But for $S' > \tilde{r}$, $\partial W(S',s)/\partial S' = -(\rho/g)S'^{\rho/g-1}(\beta + W(S',s'))/(S'^{\rho/g} - s'^{\rho/g}) < 0$. Therefore $S' > \tilde{r}$ is never optimal, as previously claimed, and the optimality conditions can be written as:

(15) $\Pi(s') = \rho W(S',s')$

(16) $\Pi(S') - \Pi(s') \geq \rho\beta$, with equality unless (w.e.u.) $S' = M$. [15]

2.4 Entry. In an equilibrium, the optimal (S',s') for an individual price-setter must coincide with the (S,s) implemented by its competitors; from the above discussion, this fixed-point property is equivalent to the requirement that the original (S,s) pair solve (15)-(16). The last equilibrium condition is that firms' intemporal profits, net of fixed costs, be zero:

(17) $\rho W(S,s) = h$, or:

(18) $\Pi(s) = h$

by (15) with $s' = s$. By (18), operating profits always cover fixed and variable cost, so firms are always willing to operate and satisfy demand.

We can now express all equilibrium conditions in terms of $E \equiv (S,s,\nu,\underline{r},\tilde{r})$ only. In order make them as clear as possible without rewriting in full the expressions for the returns to search, equilibrium profit and value from an (S',s') strategy, we shall make these functions' dependence on E explicit by denoting them as $\Gamma_E(p)$, $\Pi_E(p)$, and $W_E(S',s')$ respectively. Γ_E is defined from (3) and (1); Π_E results from (10); finally, given Π_E, W_E is given by (12).

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[15] If $S' = M$ in (16), $\Pi(S') \geq \rho(W+\beta) > 0$ requires $S' \leq \tilde{r}$, so $\tilde{r} = M = S'$.

Proposition 3: An equilibrium is a quintuple $E \equiv (S,s,\nu,\underline{r},\bar{r})$, with $s < S < +\infty$, $0 < \nu < +\infty$, and $\underline{r} \leq p^m$, which solves: [16]

$$\Gamma_E(\underline{r}) \leq \underline{\gamma}, \qquad \text{w.e.u. } \underline{r} = M. \tag{19}$$

$$\Gamma_E(\bar{r}) \leq \bar{\gamma}, \qquad \text{w.e.u. } \bar{r} = M. \tag{20}$$

$$\Pi_E(s) = h. \tag{21}$$

$$\Pi_E(S) \geq h + \rho\beta, \qquad \text{w.e.u. } S = M. \tag{22}$$

$$W_E(S,s) = h. \tag{23}$$

This system of five equations in five unknowns summarizes the interdependent decisions (entry, price strategies and search) of the many firms and different types of buyers in the market. The fixed-point nature of an equilibrium is apparent from the fact that E enters the functions in the system as both parameter and argument.

3 - EQUILIBRIUM

We first examine the existence and properties of an equilibrium in a market with identical buyers. Its clear-cut results show that inflation in a second-best world can in fact improve welfare, and provide intuitions which will be useful in the more difficult case of heterogeneous buyers.

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[16] That $\underline{r} \leq p^m$ is required in Proposition 2 for quasiconcavity. The constraints $s \geq c$, and $S \leq \bar{r}$ are implied by (21) and (22) respectively. The same system describes the equilibrium for a market without entry, if one fixes ν and lets h vary instead; such an equilibrium may fail to exist, i.e. (19)-(23) with a given ν may entail $h < 0$.

3.1-Identical buyers. Let buyers have the same search cost $\bar{\gamma} = \underline{\gamma} \equiv \gamma$ and unit monopoly demand, i.e. $V(p) = \max(M-p, 0)$. Since they have the same reservation price $\underline{r} = \bar{r} \equiv r$, equilibrium demand is:[17]

$$(24) \qquad D(p) = 1/\nu \quad \text{for } p \leq r, \quad D(p) = 0 \text{ for } p > r$$

Since demand is inelastic below r and zero above, firms optimally set $S = r$.[18] Equation (20) then becomes: $S - \int_s^S p \, dF(p) = \gamma$, or:

$$(25) \qquad S = \min \{M, \gamma + (S - s)/\mathrm{Ln}(S/s)\}.$$

Price-setting is thus constrained by the minimum of consumers' willingness to pay and of the average price in the market plus the search cost. This is the model analyzed in detail in Benabou [1988].

Theorem 1: If buyers have the same search cost and inelastic demand, there is a unique equilibrium. A higher inflation rate increases price dispersion but reduces real prices S and s and the number of firms ν. Inflation improves consumer surplus and (at least when ρ is not too large) social welfare.

For proofs and details, see Benabou [1988]. The intuitions behind these results are simple. In equilibrium buyers accept the first price offered, but search matters as a credible threat; a decrease in γ reduces real prices S and s, price dispersion S/s and the number of firms ν. Indeed, the equilibrium varies continuously with γ from the monopolistic end (S = M and prices independent of search costs, as in Diamond [1971]) to the competitive (Bertrand, S = s = c) end of the spectrum. As to a higher rate of inflation, it increases

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[17] Formally, replace V(p) by M - p in (10) and use l'Hopital's rule to show that $(\bar{r} - \underline{r})/(\bar{\gamma} - \underline{\gamma})$ tends to $\Gamma'(r) = z(r)F(r) = 1$ as $[\bar{\gamma}, \underline{\gamma}]$ shrinks to $\{\gamma\}$ and $\bar{r}$ and $\underline{r}$ tend to a common limit r, with $\Gamma(r) = \gamma \leq \Gamma(M)$.

[18] Formally, one can show that (21) and (23) now prevent equality in (22).

price dispersion and the returns to search; this leads to more competitive pressure on firms, a lowering of the whole (S,s) price range and an increase in consumer surplus. Due to staggering, firms' aggregate net profits per unit of time equal a single firm's net average profits over the cycle; when ρ is small, this is close its net discounted profits over the cycle, which are zero due to entry. Thus total welfare increases as well.

3.2 Heterogeneous buyers. Let us revert to different buyers with general preferences satisfying Assumption 1, and simplify the problem by assuming that $\gamma = 0$. The first condition in (19) then easily gives $\underline{r} = s$, and quasiconcavity thus requires $s \leq p^m$. Condition (20) is unchanged and determines $\bar{r} \leq M$ as a function of (S,s). Condition (21) gives the equilibrium density of firms:

$$(27) \qquad \nu = \frac{\pi(s)}{h\bar{\gamma}}[V(s) + \bar{\gamma} - \bar{V}(S,s)]$$

which must be finite and positive. Substituting ν in (22) and (23) then leads to a system in the two unknowns (S,s) only, which fully determines the equilibrium, namely (let $\delta \equiv \rho/g$):[19]

$$(28) \qquad \pi(S)[V(S) + \bar{\gamma} - \bar{V}(S,s)] \geq \left[1 + \frac{\rho\beta}{h}\right]\pi(s)[V(s) + \bar{\gamma} - \bar{V}(S,s)],$$

w.e.u. $S = M$;

$$(29) \qquad \int_s^S \frac{\pi(u)[V(u) + \bar{\gamma} - \bar{V}(S,s)]}{\pi(s)[V(s) + \bar{\gamma} - \bar{V}(S,s)]} u^{\delta-1} du = \left[\frac{g\beta S^\delta}{h} + \frac{S^\delta - s^\delta}{\delta}\right]$$

provided $s \leq p^m$ and the right-hand-side of (28) is positive. Using in particular the strict quasiconcavity of Π (Proposition 2) and the properties of z(p) from Assumption 1, we prove the following result:

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[19] These conditions assume $\delta > 0$ and $h > 0$. Similar ones hold when there is no discounting or no fixed costs. When $\rho \longrightarrow 0$, $(S^\delta - s^\delta)/\delta$ tends to $\mathrm{Ln}(S/s) = T/g$, and firms maximize profits per unit of time, which is the limit of ρW, as given by (11). When $h = 0$, (21) implies that $s = c$.

Theorem 2: When $\gamma = 0$, there exists an equilibrium; buyers search actively and each firms' sales are cyclical.

Proof: in appendix.

This result bears some relation to that of Caplin and Nalebuff [1988]. In both cases, the existence of equilibrium in a market with imperfect price competition obtains because the demands of buyers with appropriately distributed individual characteristics aggregate to quasi-concave profit functions, and this in turn ensures that firms' best-reply correspondence has a fixed point. Caplin and Nalebuff deal (with great generality) with static competition in differentiated products and heterogeneity in consumer preferences; here we deal with dynamic price strategies and sequential search for a single good. The necessary counterpart (in terms of tractability) of these more complex strategies of buyers and sellers is a more restrictive distribution of consumer characteristics: fairly general but identical preferences, and uniformly distributed search costs (Assumptions 1 and 2).

With heterogeneous consumers, the uniqueness of the equilibrium can generally not be established analytically, but all numerical simulations strongly suggest that such is the case. [20] In any case all equilibria share the same basic features: buyers with low search cost actively seek firms which have not revised their nominal price recently -and are thus at a low point of their (S,s) cycle. Buyers whose search cost is higher search less and do not fare as well. Correspondingly, a firm's sales increase over the time during which its nominal price remains fixed, and fall after each adjustment.

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[20] When $z(p) \equiv 1$ and $h = \rho = 0$, one can show that the equilibrium is unique. S is a non-decreasing function of $\bar{\gamma}$, while $s = c$ and the inflation rate and price adjustment costs only affect (negatively) the density of firms ν.

4 - INFLATION, OUTPUT, THE COST OF SEARCH, AND WELFARE

4.1-Price dispersion, resource costs and social welfare. As mentioned earlier, there is substantial empirical evidence that higher rates of inflation are associated with greater price dispersion. On the other hand, Cechetti [1986] and Kashyap [1986] identify markets where increases in inflation are accompanied by more frequent but not larger price adjustments: T decreases, but S/s remains constant and may be quite small - e.g. 2 or 3 percent. These authors view this as evidence against fixed menu costs and appeal to costs which increases in the size of the adjustment. While the (S,s) model is no doubt an abstraction, we shall show that it is not at all inconsistent with such observations, especially in an equilibrium context.

More generally we now examine, in the light of the model, the issues of price dispersion, the cost of induced search, the long-run Phillips curve, and the effects of inflation on the different components of welfare. We define welfare as the sum of aggregate consumer and producer surplus, and not in any Paretian sense. Indeed it will be clear that inflation causes substantial redistributions between firms and consumers as well as among consumers with different search costs.

We proceed in two stages. First search costs, surplus and welfare are computed as functions of the equilibrium $E = (S,s,\bar{r},\underline{r},\nu)$; this brings to light the different channels through which inflation operates. As the model is too complicated to perform comparative statics exercises analytically, simulations are then performed to assess the direction and magnitude of these different effects.

Turning to buyers first, their welfare per unit of time is simply the value of their search strategies:

$$(30) \qquad B_c = \int_{\underline{\gamma}}^{\bar{\gamma}} [V(R(\gamma)) + \Gamma(R(\gamma))]\, q(\gamma)\, d\gamma$$

which depends only on the equilibrium (S,s) bounds. In particular, if buyers have unit demands, as in Benabou [1988], Diamond [1988], or Theorem 1, all that matters is their common reservation price, i.e. the highest price in the market. A more useful decomposition of buyer welfare is obtained as the difference between the gross surplus from all transactions and the total resource cost of search C_s:[21]

$$(31) \qquad B_c = \int_s^S V(p)\, N(p)\, dF(p) - C_s.$$

where $N(p) \equiv \nu.n(p) = \min(V(p),\bar{V}(\underline{r})) + \bar{\gamma} - \max(\underline{\gamma}, V(S,s))$ is the total number of transactions at a price p, and N.dF sums to one on [s,S]. A buyer with cost γ searches on average $k(\gamma) \equiv 1/F(R(\gamma))$ times, but the first one is free, so:

$$C_s = \int_{\underline{\gamma}}^{\bar{\gamma}} \gamma.(k(\gamma)-1)\frac{d\gamma}{\bar{\gamma}-\underline{\gamma}} = \int_{\underline{r}}^{\bar{r}} \Gamma(r)\left[\frac{1}{F(r)} - 1\right]\Gamma'(r)\frac{dr}{\bar{\gamma}-\underline{\gamma}}, \qquad \text{or:} \quad [22]$$

$$(32) \qquad C_s = \int_{\underline{r}}^{\bar{r}} z(r)\Gamma(r)\frac{dr}{\bar{\gamma}-\underline{\gamma}} - \frac{\bar{\gamma}+\underline{\gamma}}{2}$$

Clearly, C tends to zero as $\bar{\gamma}-\underline{\gamma}$ does, so that buyer heterogeneity is required for costly search to take place in equilibrium.[23] Also, a more desirable good (an upward shift in z(p)) will tend to increase C_s. In (31), inflation will affect surplus (and output) both through F, the <u>distribution of prices</u> in the market, and - via search decisions- through N, the

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[21] The equality of (30) and (31) folllows from (32) and the definition of Γ.

[22] This equation assumes for simplicity that $\bar{r} < M$. If not, the second term must be replaced by $(\Gamma(M)^2 - \underline{\gamma}^2)/(2(\bar{\gamma} + \underline{\gamma}))$.

[23] Each buyer's labor endowment L should also cover his search costs. This cannot be ensured with probability one, but the probability that a buyer runs out of resources before finding $p \leq R(\gamma)$ is very small if $L >> \gamma[1/F(R(\gamma))-1]$. For all $r \geq s$, (3) implies $\Gamma(r) \leq (V(s) - V(r))F(r)$, so that $\gamma/F(R(\gamma)) \leq V(s)$ and it suffices to assume that $L >> V(c)$.

distribution of transactions across prices. This latter effect is accompanied by a change in the total amount of search and its cost C_s, given by (32).

In the absence of discounting, B_c would represent total social welfare. Indeed, as firms' price revisions are uniformly staggered, aggregate profits per unit of time coincide with a given firm's average profits over the (S,s) cycle, times the number of firms ν; when $\rho = 0$, these average profits, net of fixed and menu costs, are equalized by entry to zero (let $\rho \to 0$ in (17)).

When $\rho > 0$, undiscounted profits over the cycle are larger than discounted profits; since it is the latter (net of fixed costs) which are zero, total profits per unit of time are positive, and can be decomposed into:

$$(33) \qquad B_f = \int_s^S (p-c)z(p)N(p)dF(p) - \nu.\left[h + \frac{\beta}{T}\right]$$

The first term is operating profits from the N(p) transactions at each firm; the second one is total fixed costs; the last one is total adjustment cost, since a proportion $1/T = g/Ln(S/s)$ of firms adjust per unit of time.[24] Summing up, social welfare equals aggregate gains from trade, minus the resources spent on the three types of market frictions: search, price adjustment, and fixed operating costs:

$$(34) \qquad B = \int_s^S [(p-c)z(p) + V(p)]\, N(p)\, dF(p) - C_s - \nu\left[h + \frac{\beta}{T}\right].$$

The first term reflects the allocative role played by prices through production and consumption decisions. With unit consumer demand it becomes a constant. In general, however, the equilibrium (S,s) price cycles can either

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[24] Alternatively, each firm's intertemporal profits, starting from s, are zero, but they are positive until it reaches s from its initial position, i.e. until its first adjustment. Thus (equality with (33) rests on (1) and (23)):

$$B_f = \nu \int_s^S \int_0^{\Theta(p)} [\Pi(pe^{-gt}) - h]e^{-\rho t}\, dt, \text{ where } \Theta(p) \equiv Ln(p/s)/g.$$

worsen or alleviate the inefficiency of monopolistic pricing, and the latter will thus be affected by inflation.

The second term reflects the resources spent on search for better prices, which will also vary with inflation. It vanishes in models with identical buyers, since all accept the first offer received.[25] The third effect of inflation will be to tend to reduce the number of firms in the market (essentially by forcing them to charge less profitable real prices), and with it the inefficient duplication of fixed costs which characterizes monopolistic competitition when $h > 0$. Finally, this induced exit will partially offset the increase in adjustment costs per operating firm resulting from higher inflation (last term).

4.2 Simulations. To get a sense of the direction, magnitude and net balance of these four welfare effects, simulations were performed with two specifications of preferences, corresponding to isoelastic ($z(p) = p^{-\alpha}$, $M = +\infty$) and linear ($z(p) = M - p$) monopoly demand functions. The inflation rate g ranges from zero to fifty per cent a year. A very large number of parameter values were explored, but only a dozen typical outcomes are illustrated, in Figures 1a to 7c. The reference set of parameters are $\chi = 0$, $\tilde{\gamma} = .10$, $\beta = h = 10$, $\rho = .05$, $\alpha = 4.1$ (isoelastic case) or $M = 8$ (linear case); deviations from this case are indicated on the graphs.

The first three results below are quite robust, holding across all specifications of preferences and parameter values.

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[25] In Diamond [1988], however, there is a cost of involuntary search, which is also affected by inflation: since the arrival of offers is a Poisson process, buyers, who are impatient, must still wait for the first one. The arrival rate of buying opportunities depends on the stock of goods produced but yet unsold, and the latter is determined by a zero-profit condition; it is as if the first term of (34) were multiplied by an increasing function of ν.

Result 1: As the rate of inflation increases, so does real price dispersion, although prices are changed more frequently. [26]

These results, illustrated in Figures 1a and 1b, shed light on some of the empirical evidence mentioned previously. For the isoelastic specification, both price dispersion and its rate of increase with inflation are quite significant: for g equal to 1 %, 10 % and 50 %, S/s - 1 is respectively 9.3 %, 19.5 % and 31.7 %. For the linear specification, on the contrary, price dispersion remains very small for all inflation rates: it is 1.9 % at g = 1% and only 2.8 % at g = 50 %. Conversely, the decrease in T is much faster with the second specification than with the first: for g = 1 %, 10 % and 50 %, prices are revised every 97.0, 21.2 and 6.6 months respectively in the first case, and every 20.6, 5.7 and 0.7 months in the second. Depending on market characteristics, equilibrium (S,s) rules are thus quite capable of generating a strong relationship between inflation and price dispersion (Fischer [1981], Domberger [1987], Dantziger [1987b]) or an insignificant one (as in some of the data of Cechetti [1986] and Kashyap [1986]). They are also consistent with small adjustments (and dispersion) at high inflation rates, once one recognizes that the intensity of competition both depends on (through search) and limits price dispersion. Another factor (which could easily be incorporated into the model) may be that higher inflation leads to the adoption of new price-setting technologies which lower β.

Figures 1a and 1b provide a first depiction of the interplay of price-setting and search, by showing firms' desired price p^* and the average

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[26] Price dispersion is measured by S/s - 1. Benabou [1988] shows that the log-uniform distribution's coefficient of variation of is a monotonic function of S/s. Dantziger [1987a] proves a similar result for the distribution of purchase-weighted prices of a monopolist with isoelastic demand.

Figure 1a: Inflation and real prices

$c=1 \quad \beta=h=10 \quad \bar{\gamma}=.10 \quad \rho=.05 \quad z(p)=p^{-\alpha}, \ \alpha=4.1$

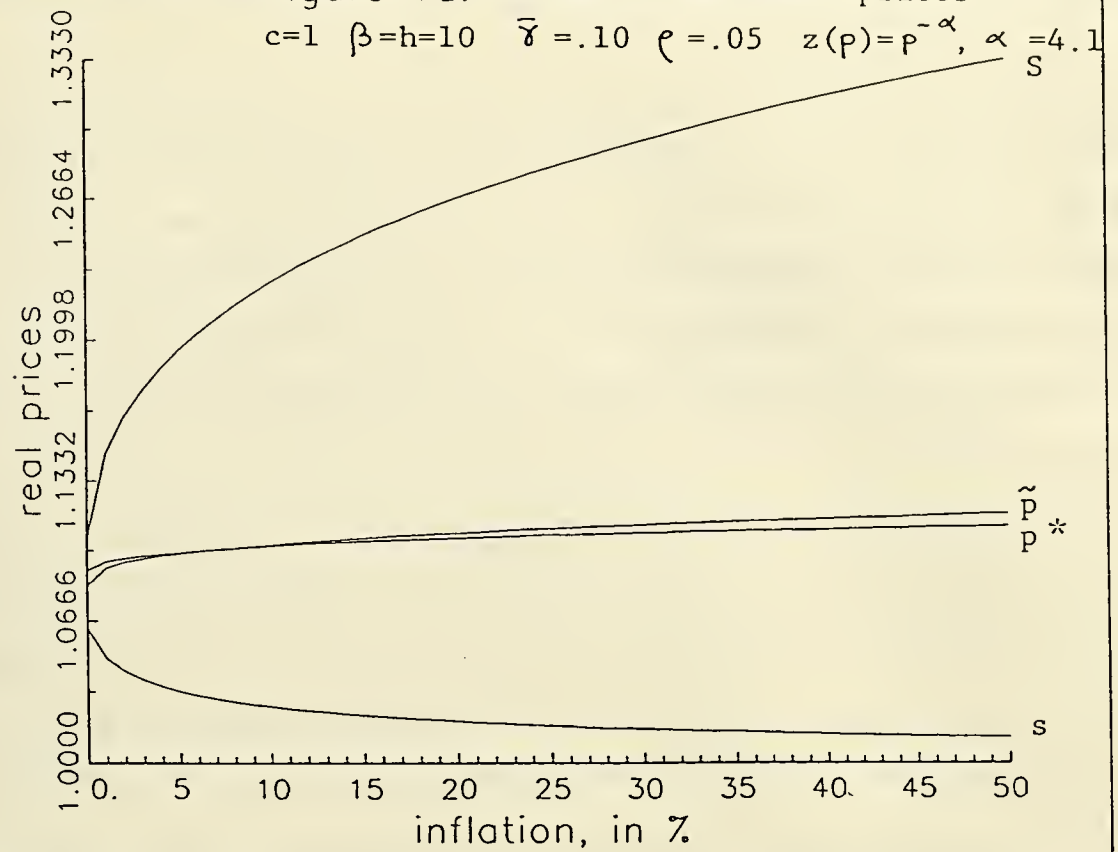
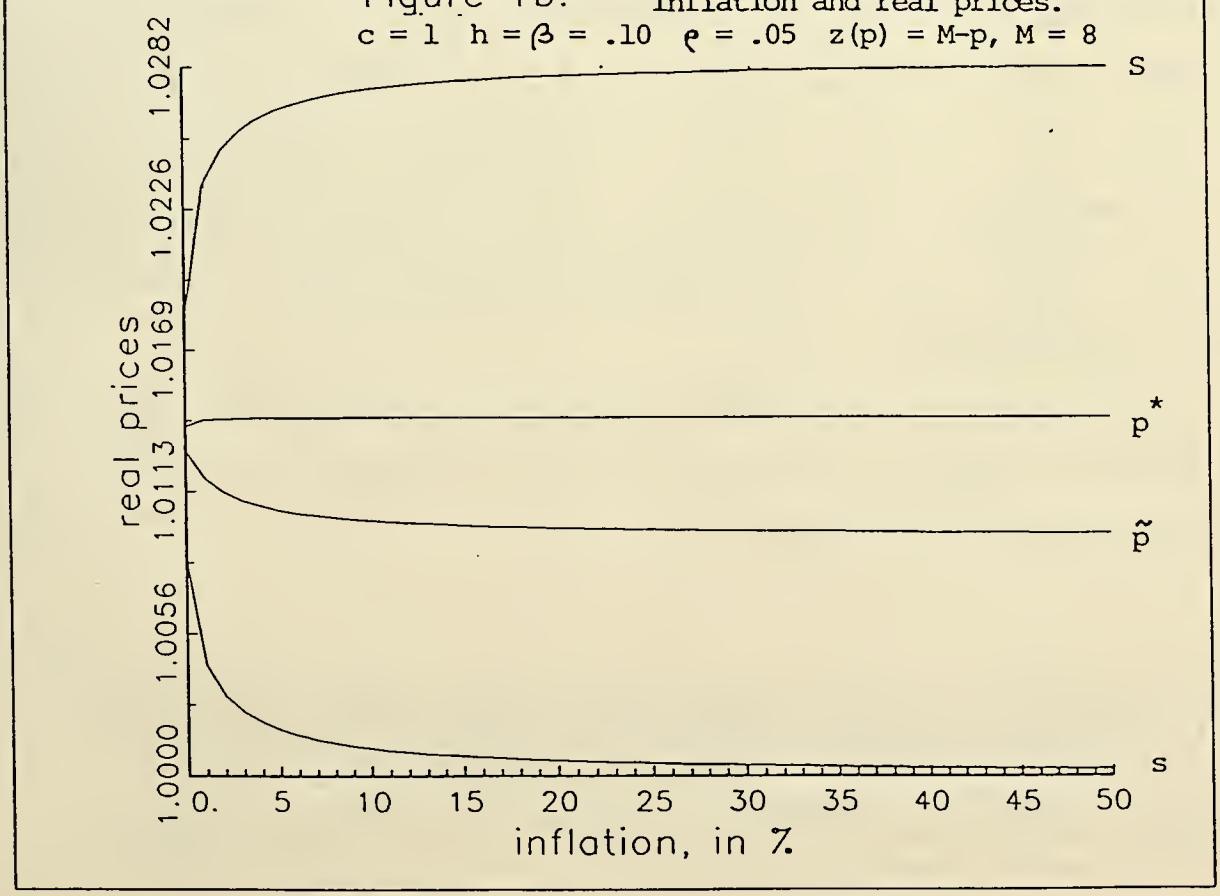


Figure 1b: Inflation and real prices.

$c = 1 \quad h = \beta = .10 \quad \rho = .05 \quad z(p) = M-p, \ M = 8$



transaction price $\tilde{p}$.[27] By definition, p^* varies inversely with the elasticity of equilibrium demand. Figure 1a implies that the elasticity of demand due to search alone decreases ($\alpha_z(p)$ is constant), while in Figure 1b it increases (p^* decreases while $\alpha_z(p)$ increases). As to $\tilde{p}$, it indicates how effective searchers are in finding the "bargains" which inflation creates in the market.

The equilibrium in the absence of inflation is also noteworthy. Let us start by asssuming zero discounting. When $g = 0$, firms adjust once and for all to the unique (by strict quasiconcavity of $\Pi(p)$) real price p^* which maximizes profits per unit of time. The outcome is thus a static, single price equilibrium, with $S = s = p^*$ and no search taking place, but where the threat of search by buyers with very low search costs ($\underline{\gamma} = 0$ and $q(0) > 0$) forces p^* well below the monopoly price p^m.[28] When $\rho > 0$, a second effect generates a discontinuity in (S,s) at $g = 0^+$: even at extremely low, but positive inflation rates, the optimal S and s remain bounded away from one another, generating price dispersion and search. This occurs because the discrete cost of adjustment β must be compensated by a discrete increase in discounted profits: (14) requires that $S > s$. Because discount rates are typically small, this effect is much less important than the first one, as seen from Figures 1a-1b.

The next result, illustrated in Figures 2a and 2b, rationalizes the conventional wisdom about the cost of inflation due to increased search, but with some important qualifiers.

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[27] In $\tilde{p}$, prices are weighted by the number of buyers who end up paying them, but not by the quantity bought. A purchase-weighted average would essentially vary inversely with output, which is examined later.

[28] See Rob [1985] for a discussion of the role played by the density of consumers with near-zero search costs. When $\bar{\gamma} = \underline{\gamma} > 0$, S and s converge to p^m, as expected, when g goes to zero (and $\rho = 0$); see Benabou [1988].

Result 2: As the rate of inflation increases, so do the total number of searches and the total resources spent on search.

The basic mechanism is clear: although inflation may cause prices to be higher or lower on average (see $\tilde{p}$), it always increases dispersion, and thereby results in increased search and more resources spent on search.[29] There is, however, an opposing force at work in equilibrium: while more price dispersion generates more search, more search intensifies price competition and allows less price dispersion.[30] This is why C_s increases rapidly with inflation rates of up to about 20 %, then remains approximately constant; above this level, the two effects almost cancel out.

The other observation to be drawn from Figures 2a and 2b is that the total cost of search is rather small. In other words, on average consumers search very few times; moreover, this result is very robust to variations in $\tilde{\gamma}$ and other parameters. Three explanations come to mind: the first one is clear from the above discussion. Secondly, the equilibrium distribution of prices has density proportional to 1/p, so that low prices are more common than high ones and thus relatively easy to find. Finally, with $\gamma = 0$ and uniformly distributed search costs, the market is rather competitive (e.g. $p^* \ll p^m$) and price dispersion never extremely large; this reduces the scope for search. While alternative distributional assumptions may well lead to a higher C_s, the first two factors will still limit its size.

The next result (and Figures 3a-3b) shows that the exit of firms constitutes a second feedback which limits the impact of inflation on real prices.

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[29] Buyers with low search cost can be shown to always search more; the behavior of those with high search costs is ambiguous, but all simulations show that the total number of searches increases with inflation, in a manner similar to the total cost of search.

[30] This interaction had been noted by Paroush [1986] in his discussion of the effects of inflation.

Figure 2a: Inflation and resources spent on search.

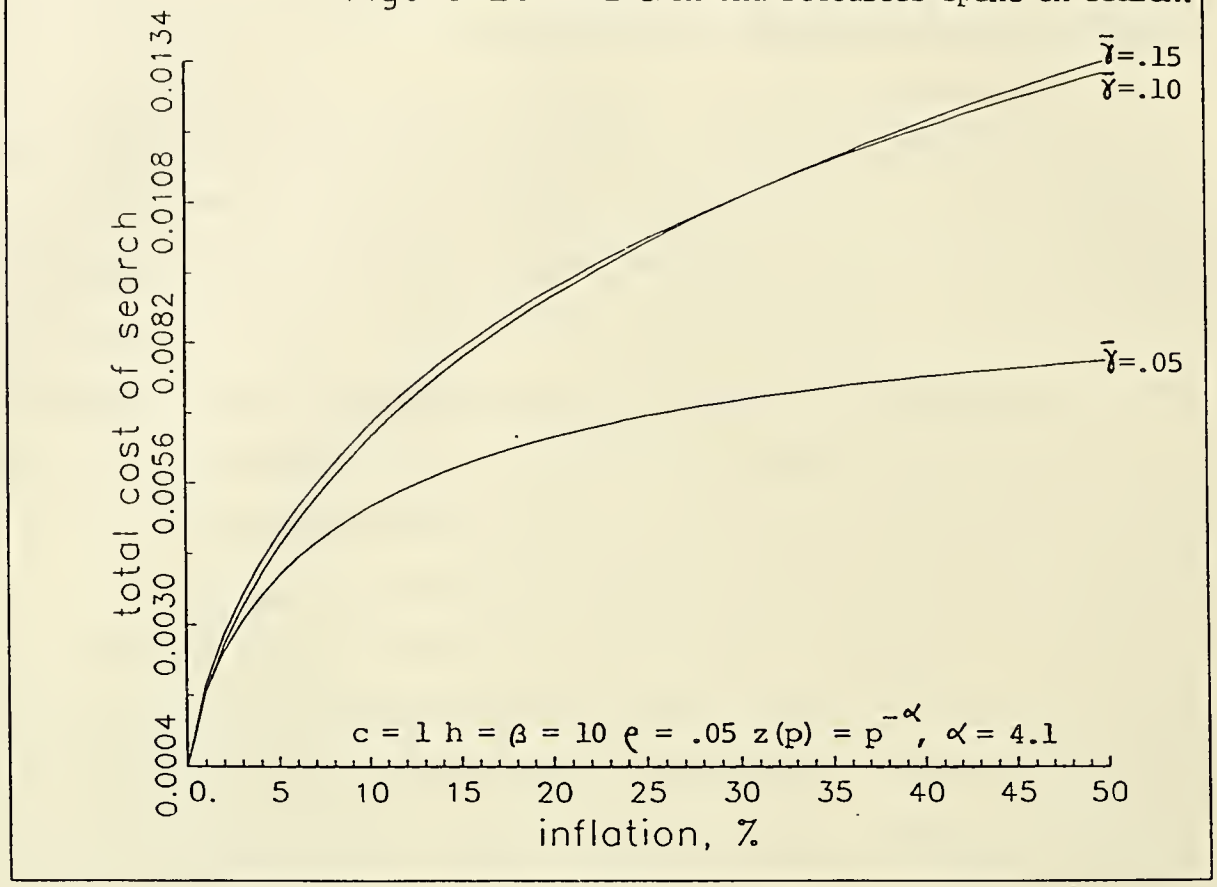
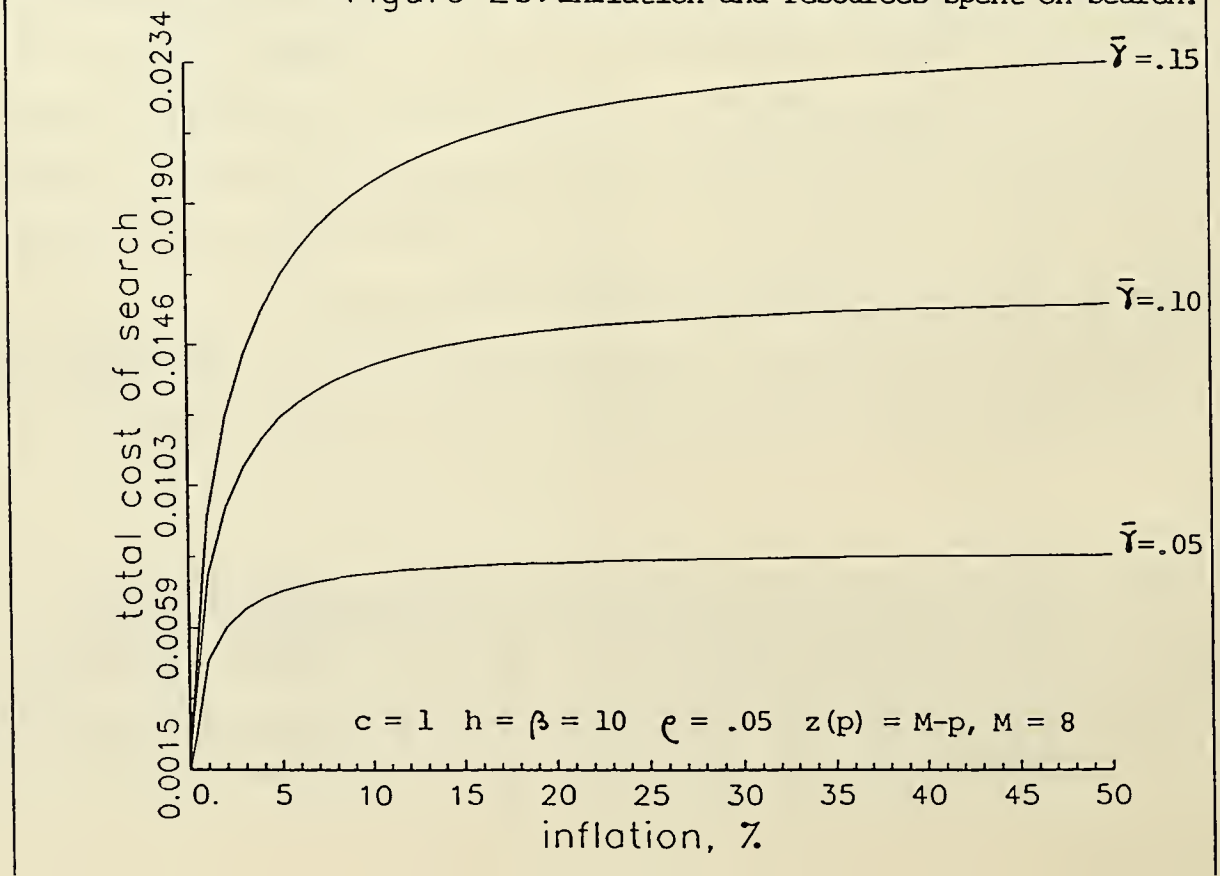


Figure 2b: Inflation and resources spent on search.



In fact, the effects of search and exit as dampening forces are visible on all equilibrium variables (most curves become quite flat above g = 20 %). Because informal arguments fail to take them into account, they are likely to overestimate both the costs and benefits of inflation.

Result 3: As the rate of inflation increases, the number of firms in the market decreases.

With entry, a firm's profits over the cycle just cover its fixed costs. Higher inflation forces it to widen its price range around p^* (Result 1); in addition, increased search (Result 2) may force p^* down; finally, it incurs more frequent menu costs (Result 1). In equilibrium, the negative impact of these effects on profits is offset by the exit of some firms, which leaves the remaining ones with a larger market share.[31] Because their adjustment costs are now smaller compared to their potential sales, it is now worthwile for them to revise their prices even more often, and this constitutes the second limiting force on price dispersion and search.

The model's prediction that inflation results in markets which are more concentrated (but not necessarily less competitive, as will be seen below) could be tested by checking whether prolonged inflationary periods are associated with a greater numbers of bankruptcies and mergers.

The elimination of firms from a monopolistically competitive market is efficiency-improving, because it reduces rent dissipation (the duplication of fixed and pricing costs). On the other hand, bankruptcies are likely to entail economic (reorganization, unemployment) or political costs which might realistically offset this efficiency gain from inflation, at least from an elected policy-maker's point of view.

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[31] Conversely, inflation reduces net profits in a market with a fixed number of firms.

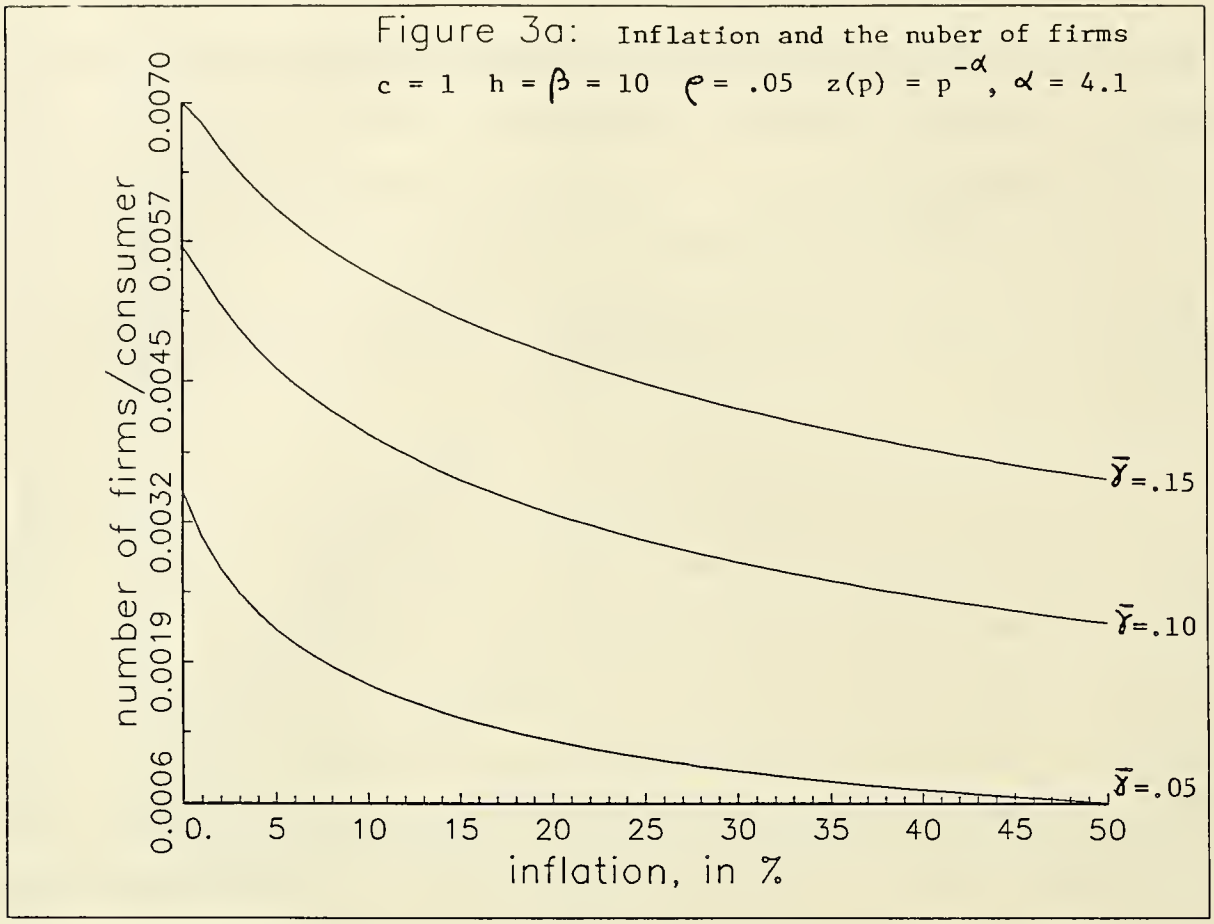


Figure 3a: Inflation and the nuber of firms
c = 1 h = β = 10 ε = .05 z(p) = p^-α, α = 4.1
number of firms/consumer
0.0006 0.0019 0.0032 0.0045 0.0057 0.0070
0. 5 10 15 20 25 30 35 40 45 50
inflation, in %
γ̄ =.15
γ̄ =.10
γ̄ =.05

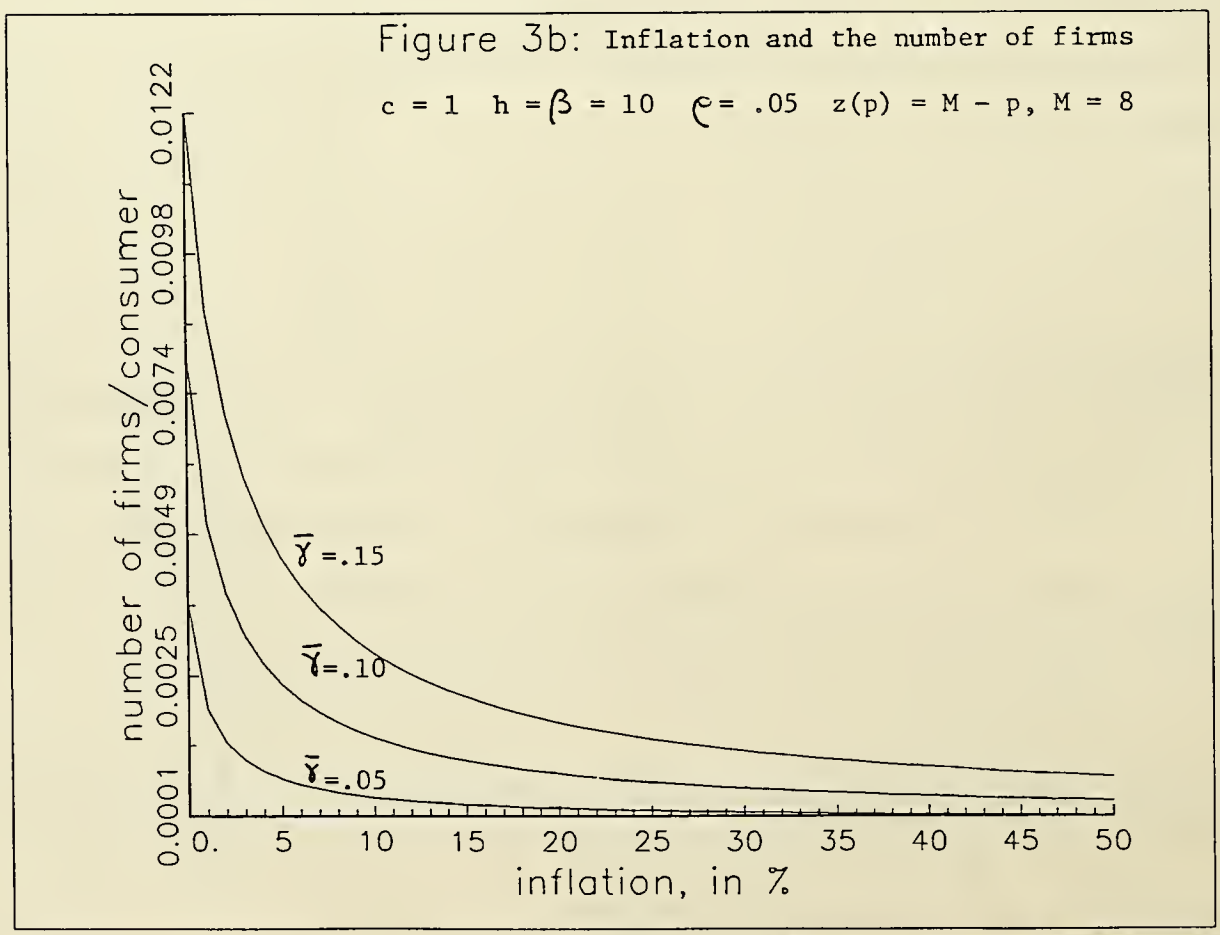


Figure 3b: Inflation and the number of firms
c = 1 h = β = 10 ε = .05 z(p) = M - p, M = 8
number of firms/consumer
0.0001 0.0025 0.0049 0.0074 0.0098 0.0122
0. 5 10 15 20 25 30 35 40 45 50
inflation, in %
γ̄ =.15
γ̄ =.10
γ̄ =.05

We now turn to the effects of inflation on output (or on the average transaction price $\tilde{p}$) and on the different components of welfare; unlike the preceding ones, they are very sensitive to the specification of consumers' preferences and the values of the market frictions $\tilde{\gamma}$, β and h.

Naish [1986] and Konieczny [1989] examined the effect of inflation on the average output of an (S,s) monopolist with zero discount rate. This effect depends on the skewess of the (log) profit function, which determines the rates of change of S and s with g, and on the curvature of the demand function, which determines the output consequences of these changes. With a constant marginal cost, if the demand is isoelastic the first effect tends to decrease output, and the second to increase it; with linear demand both effects increase output.

In a monopolistically competitive market, however, demand and profit functions are endogenous, because they depend at least on competitors' prices, if not also on the number of competitors and on buyers' strategies, as here. Moreover, these functions are themselves affected by inflation, in a manner which depends on the reactions of all sellers and buyers, and therefore on market structure (here search, entry and adjustment costs) as much as preferences. Thus not only can skewnness and curvature not be postulated, but they lose most of their relevance.

Figures 4a-4b and 5a-5b indeed show that the monopoly results need no longer hold, although they still provide a reasonable "first guess". With the isoelastic specification, the effect of inflation on output can be quite strong: a rise in g from zero to 5% a year can permanently reduce output by 4.0% ; from 5% to 10% a further decrease of 3.5% takes place (Figure 4a for $\tilde{\gamma} = .15$). With a lower value of α, the slope of the Phillips curve becomes slightly positive: for $\alpha = 1.1$, a 10% inflation rate raises output by .5% (Figure 5a). With the linear specification, the impact of inflation is most

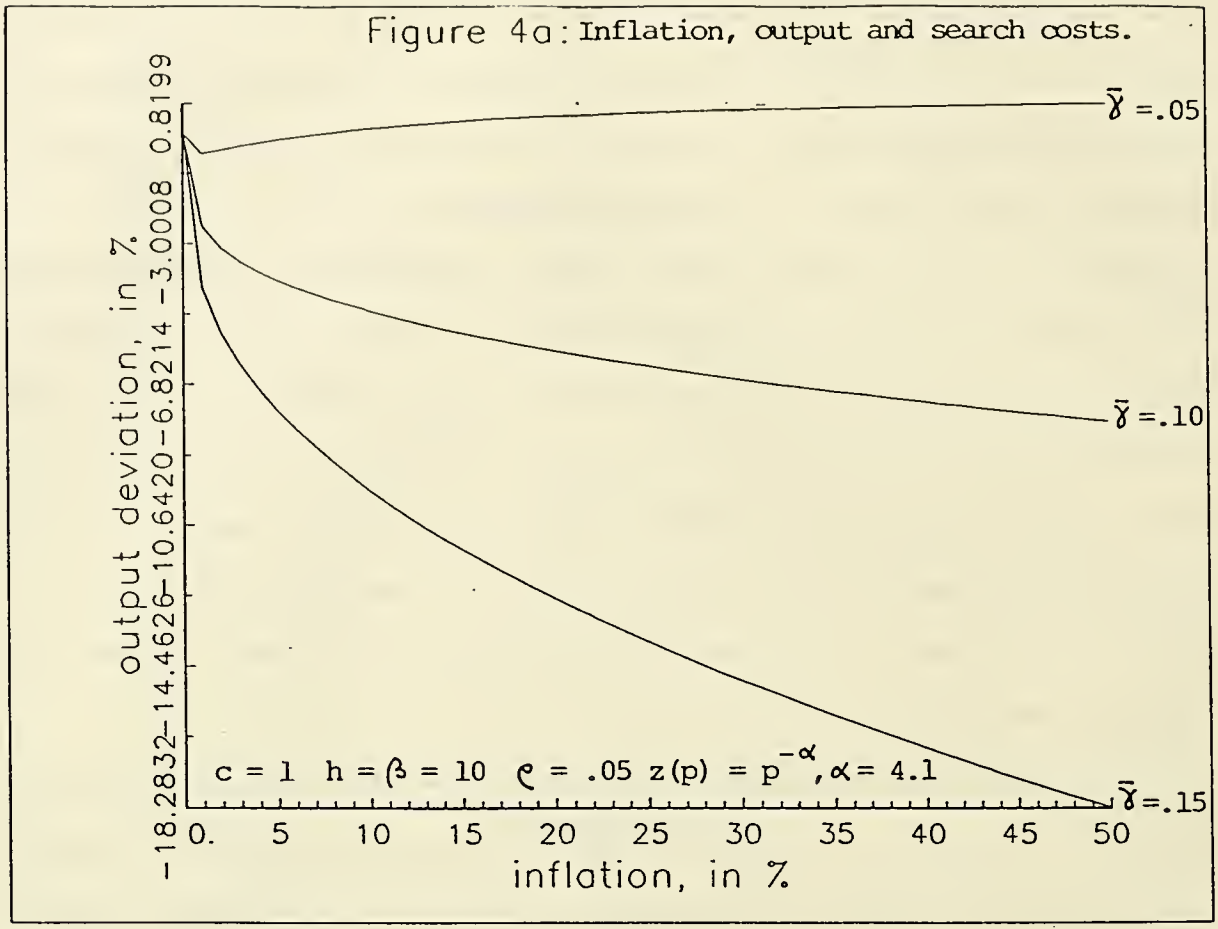


Figure 4a: Inflation, output and search costs.
output deviation, in %
0.8199 -3.0008 -6.8214 -10.6420 -14.4626 -18.2832
$\bar{\gamma}$ =.05
$\bar{\gamma}$ =.10
$\bar{\gamma}$ =.15
c = 1 h = β = 10 ϱ = .05 z(p) = $p^{-\alpha}$, α = 4.1
0. 5 10 15 20 25 30 35 40 45 50
inflation, in %

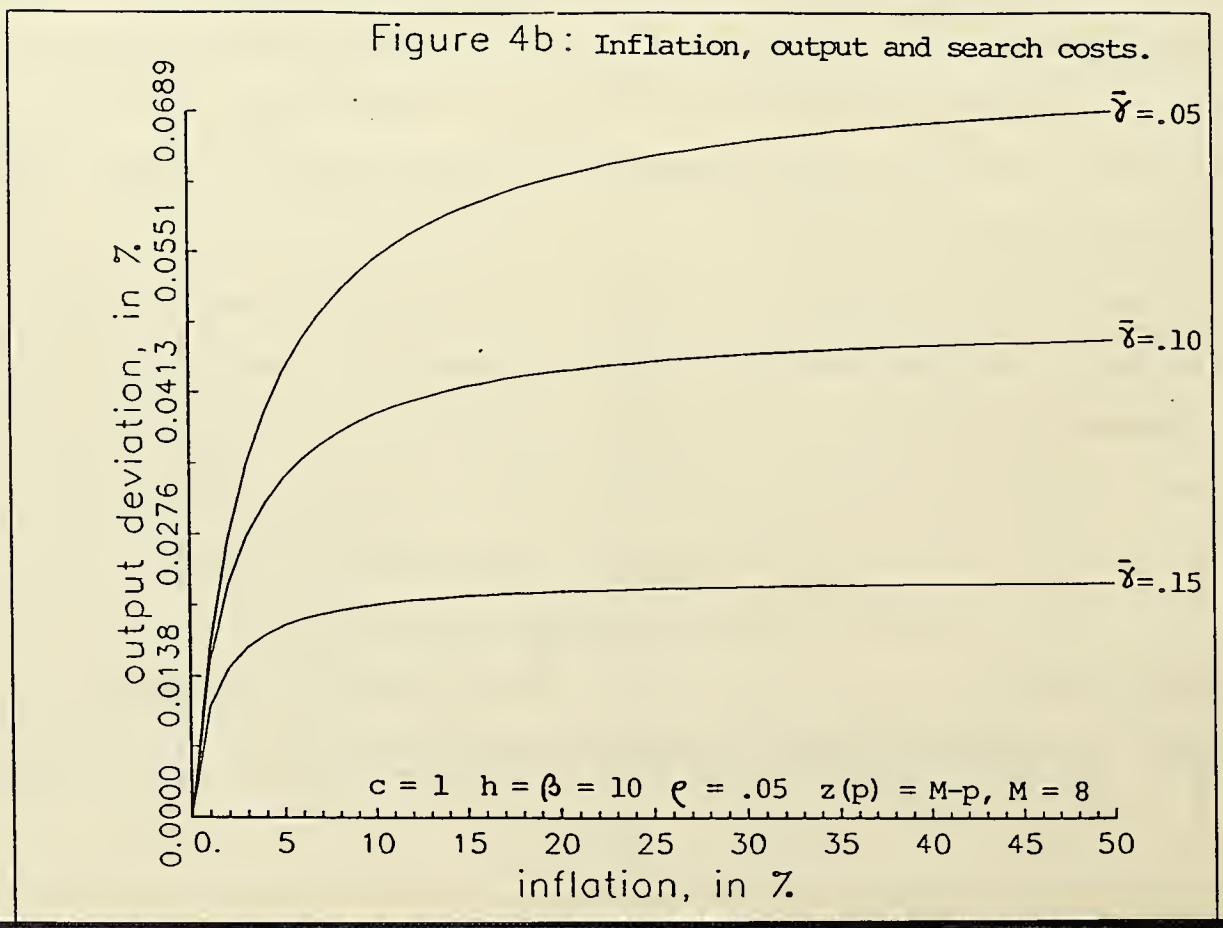


Figure 4b: Inflation, output and search costs.
output deviation, in %
0.0689 0.0551 0.0413 0.0276 0.0138 0.0000
$\bar{\gamma}$ =.05
$\bar{\gamma}$=.10
$\bar{\gamma}$=.15
c = 1 h = β = 10 ϱ = .05 z(p) = M-p, M = 8
0. 5 10 15 20 25 30 35 40 45 50
inflation, in %

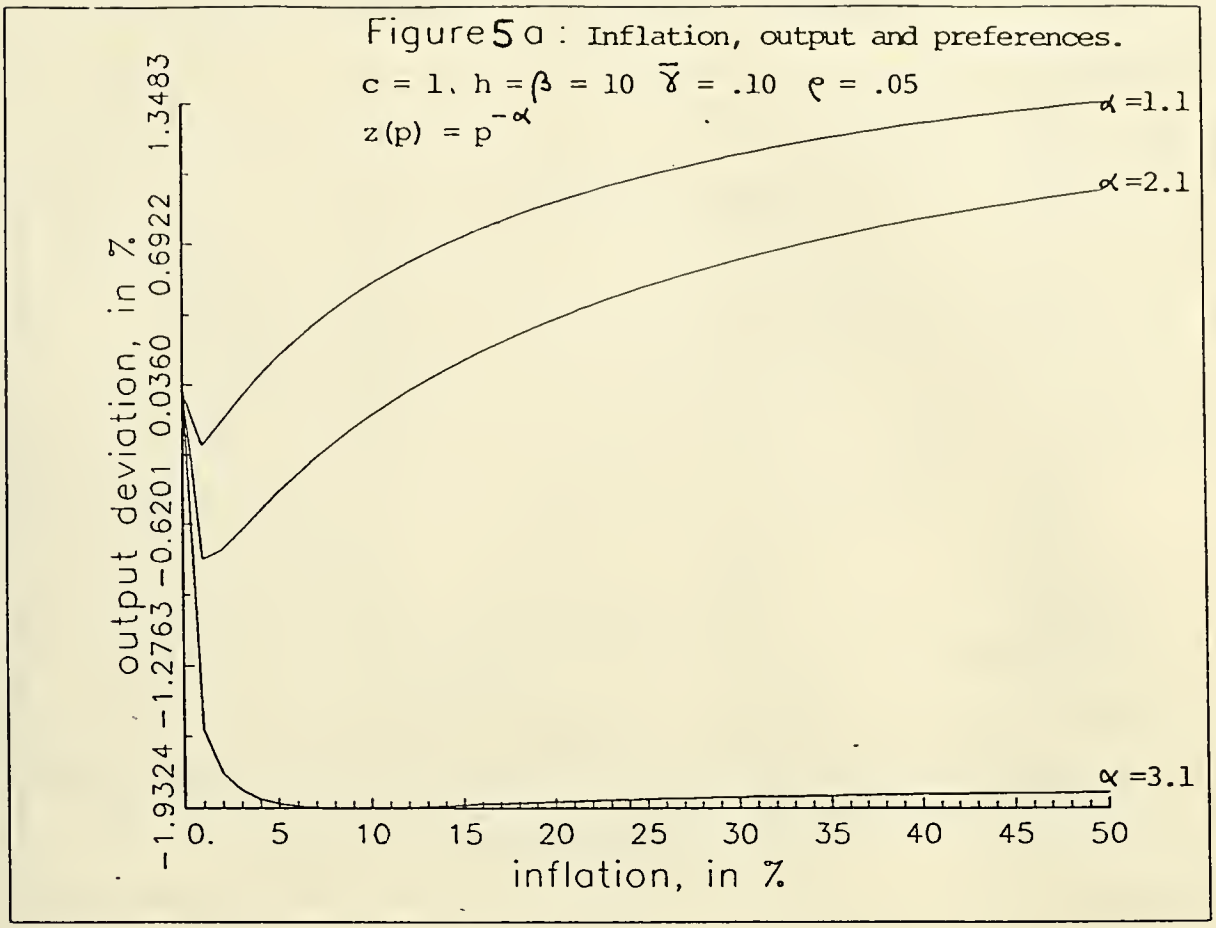


Figure 5 a : Inflation, output and preferences.
c = 1, h = β = 10 γ̄ = .10 ϱ = .05
z(p) = p^-α
α =1.1
α =2.1
α =3.1
output deviation, in %
-1.9324 -1.2763 -0.6201 0.0360 0.6922 1.3483
0. 5 10 15 20 25 30 35 40 45 50
inflation, in %

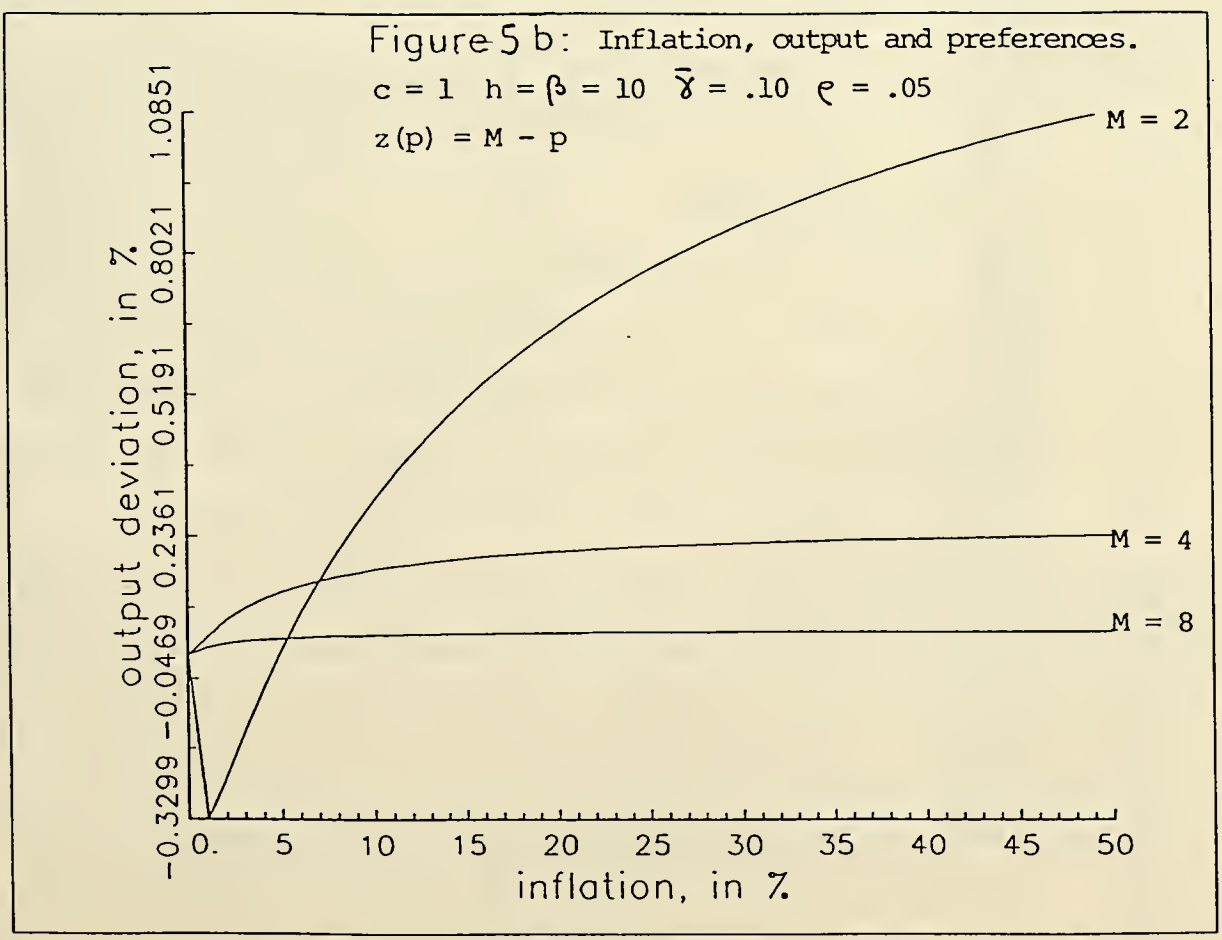


Figure 5 b: Inflation, output and preferences.
c = 1 h = β = 10 γ̄ = .10 ϱ = .05
z(p) = M - p
M = 2
M = 4
M = 8
output deviation, in %
-0.3299 -0.0469 0.2361 0.5191 0.8021 1.0851
0. 5 10 15 20 25 30 35 40 45 50
inflation, in %

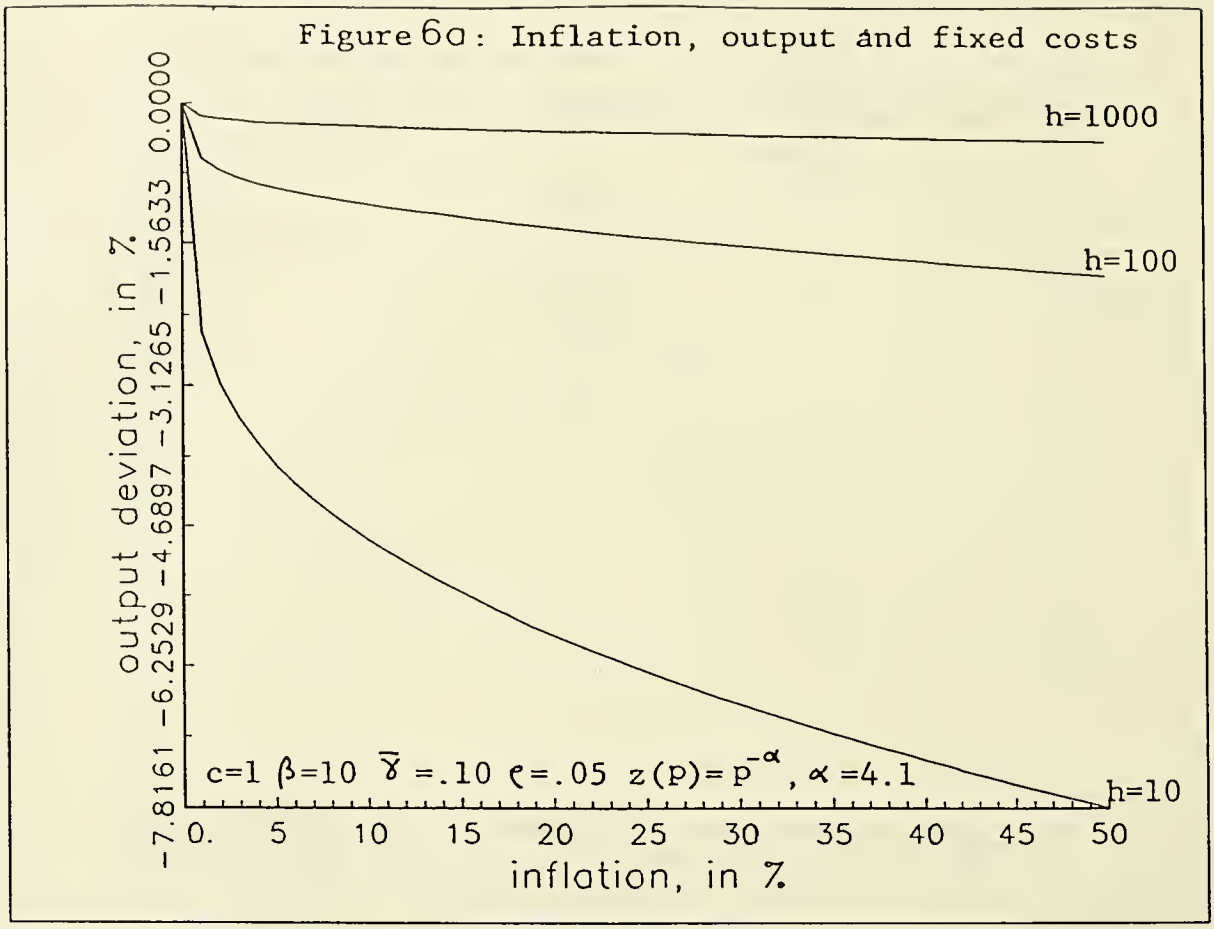


Figure 6a: Inflation, output and fixed costs
h=1000
h=100
h=10
output deviation, in %
-7.8161 -6.2529 -4.6897 -3.1265 -1.5633 0.0000
c=1 β=10 γ̄ =.10 ϱ=.05 z(p)= p^-α, α =4.1
0. 5 10 15 20 25 30 35 40 45 50
inflation, in %

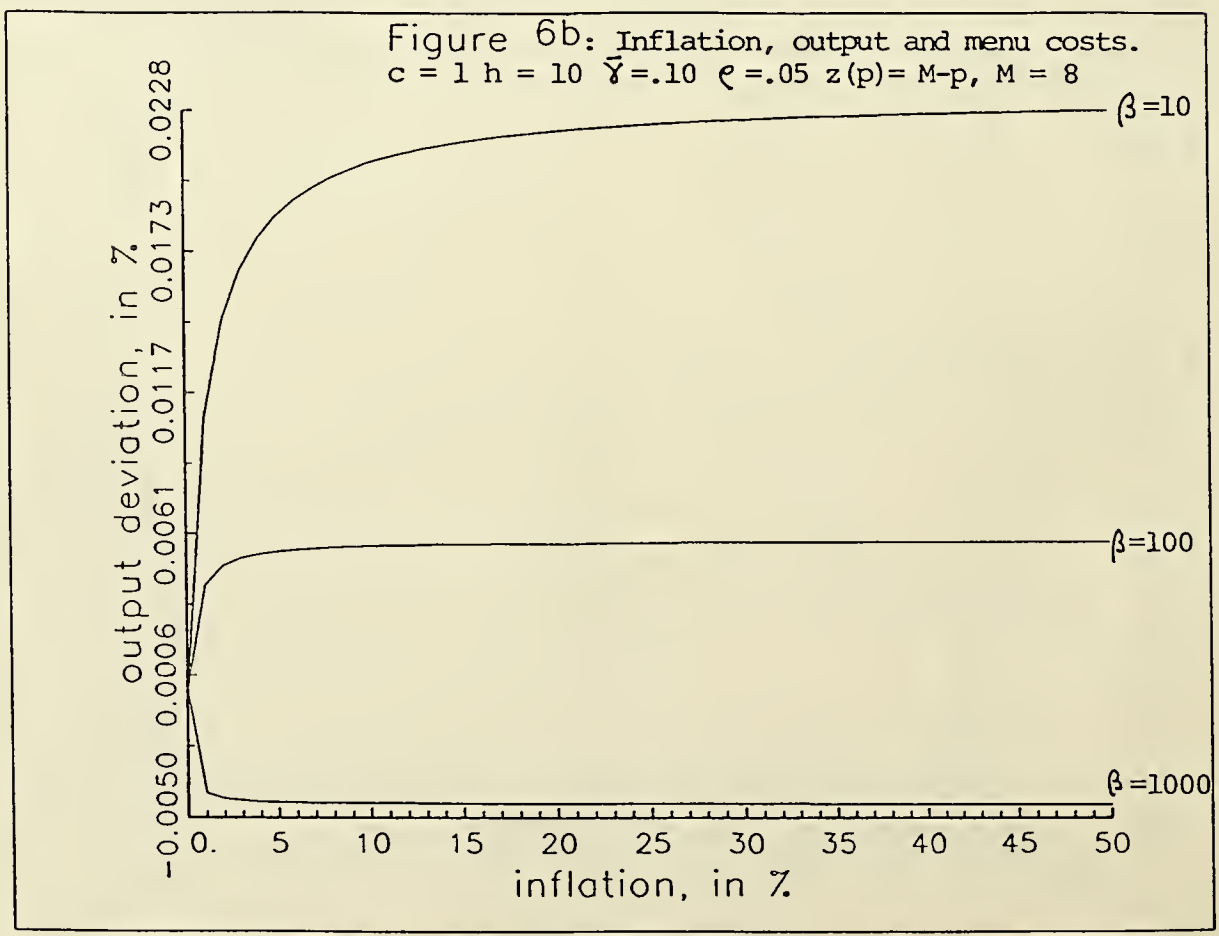


Figure 6b: Inflation, output and menu costs.
c = 1 h = 10 γ̄ =.10 ϱ =.05 z(p)= M-p, M = 8
β=10
β=100
β=1000
output deviation, in %
-0.0050 0.0006 0.0061 0.0117 0.0173 0.0228
0. 5 10 15 20 25 30 35 40 45 50
inflation, in %

often less than 1% either way (Figures 4b and 5b); as mentioned previously, inflation then mostly translates into a rapid decrease in T and ν. The simulations also bring to light an intuitive regularity:

Result 4: A higher rate of inflation can increase or decrease output. The slope of this long-run Phillips curve depends on both buyer preferences and market structure. If search costs are low enough or buyers value the good sufficiently (low α or high M), inflation tends to raise output.

Indeed, the more desired the good and the easier it is to search, the more demand (and profits) are determined by search as opposed to preferences, and the more buyers try to take advantage of increased price dispersion (Result 1) to find low prices. The model thus still delivers some positive results, in spite of the sensitivity of slope of the Phillips curve to the specification of consumer surplus. This sensitivity retrospectively justifies our modelling preferences with great generality, and should serve as a strong note of caution against drawing conclusions from macroeconomic models with specific functional forms chosen mainly for convenience.

As shown by Figure 6a, the entry cost h also significantly affects the slope of the output-inflation locus: higher fixed costs imply fewer operating firms; their larger market shares then justify more frequent price revisions, and this reduces real prices' sensitivity to inflation. The costs of price adjustment also influence (and may reverse) the slope of the Phillips curve, but in a less interpretable way (Figure 6b).

We finally turn to the analysis of welfare. Figures 7a-7b summarize the typical results emerging from many simulations. Consumer surplus essentially varies as output, and gross profits (together with the average transaction price $\tilde{p}$) in the other direction, so their dependence on inflation is ambiguous. As before, the effects are much larger with isoelastic than with

Figure 7a: Inflation and welfare

B_c+C_s = gross consumer surplus

B_c = consumer surplus, net of search costs

B = social welfare

$B-B_c$ = profits

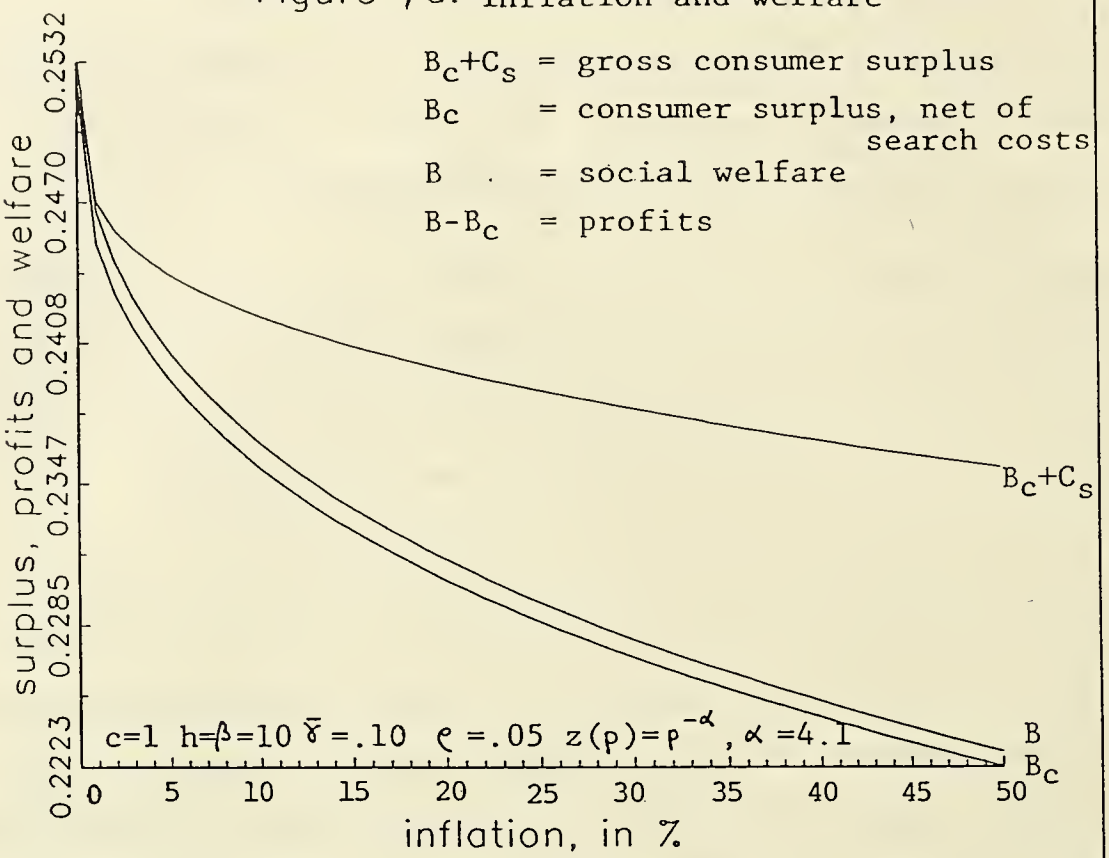
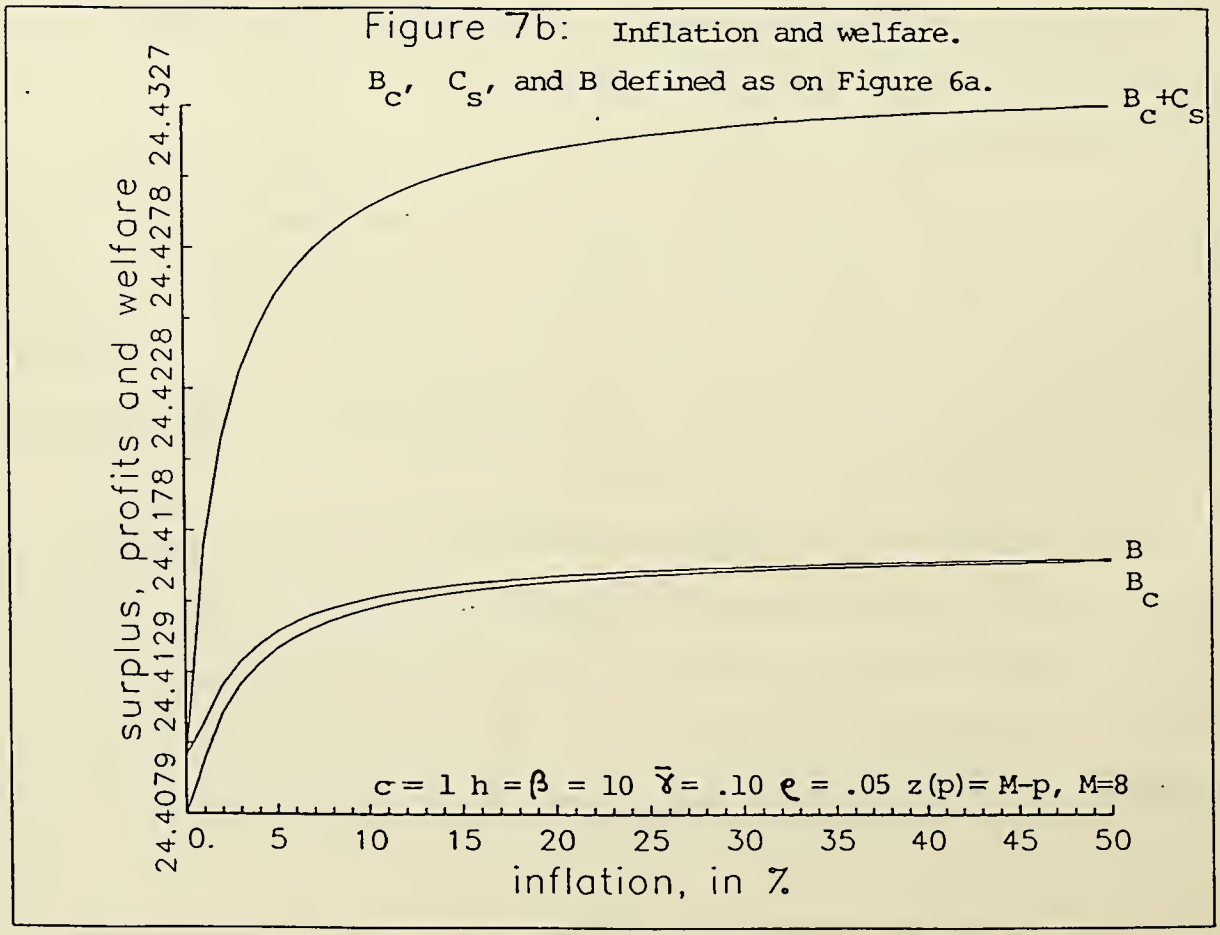


Figure 7b: Inflation and welfare.

B_c, C_s, and B defined as on Figure 6a.



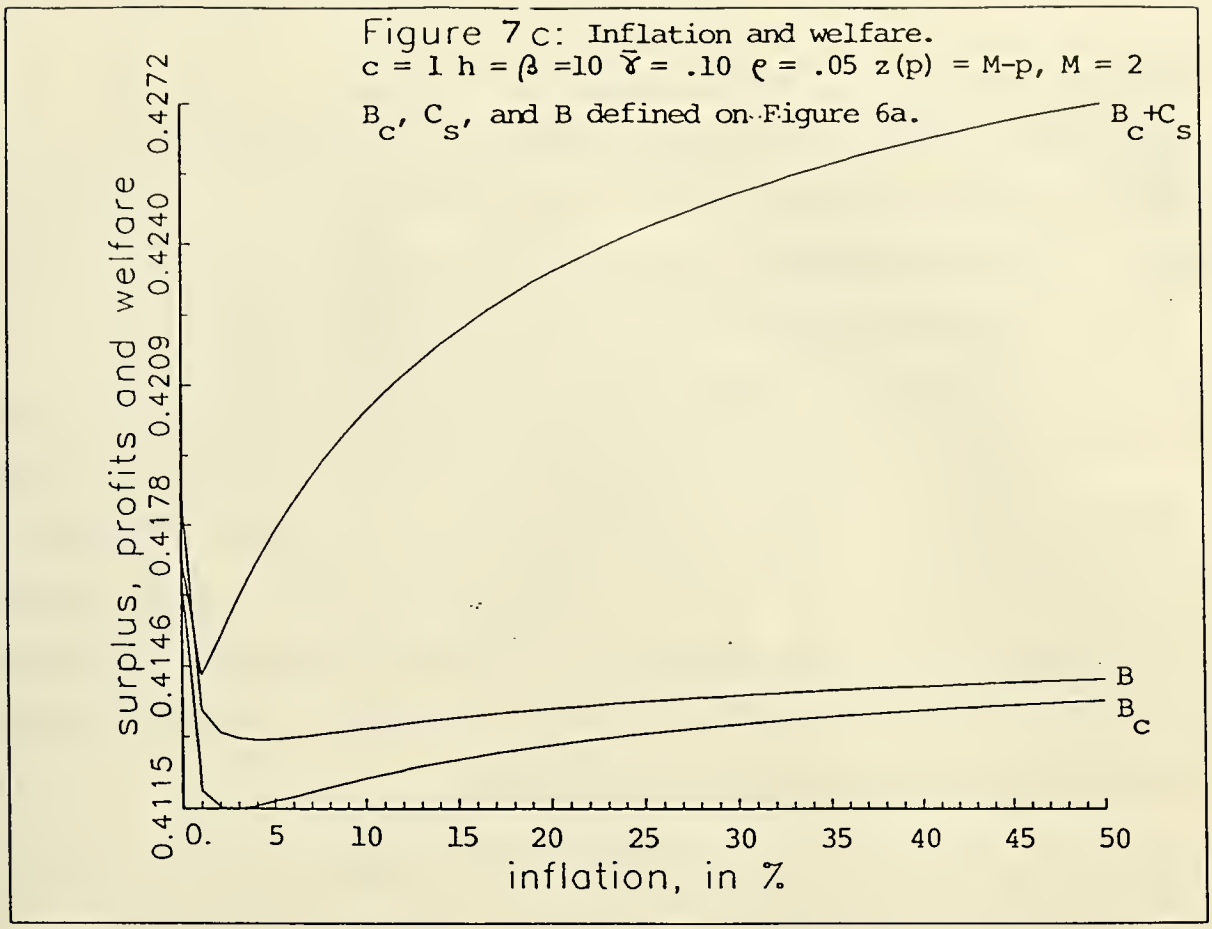


Figure 7c: Inflation and welfare.
c = 1 h = β =10 γ̃ = .10 ρ = .05 z(p) = M-p, M = 2
B_c, C_s, and B defined on Figure 6a.
B_c+C_s
B
B_c
surplus, profits and welfare
0.4115
0.4146
0.4178
0.4209
0.4240
0.4272
0.
5
10
15
20
25
30
35
40
45
50
inflation, in %

linear monopoly demand z(p). Welfare generally varies together with its main component, namely consumer surplus, as in Figures 7a-7b; its is possible, however, that increased search costs may reverse this relationship (Figure 7c), so the original concern with this cost of inflation had validity. On firms' side, the resources spent on fixed costs decrease as firms are forced out of the market, but the total cost of price adjustments increases. The sum of all these effects on firms' net profits (when $\rho > 0$) is negative, and this takes slightly away from total welfare. Again, welfare can be quite sensitive or insensitive to inflation: on Figure 7a a rise in g from 0% to 10% (resp. from 10% to 20%) decreases B by a substantial 6.6% (resp. 8.6%), but on Figure 7b it increases it by only 0.02% (resp. by 0.03%).

Result 5: Inflation may be beneficial or detrimental to social welfare, depending on buyer preferences and on market structure. The variations of welfare most often mirror those of total output, but the increased total cost of search may sometimes cause welfare to fall in spite of rising output.

Conclusion

This paper has analyzed the functioning of a monopolistically competitive search market under inflation. It has provided the first formalization of the idea that inflation increases the amount of resources spent on search, and more generally has developed a framework to rigorously examine several channels through which inflation affects competition, output and welfare.

While inflation increases price dispersion and thereby the amount and total cost of search, it also has a beneficial long-run effect by reducing the number of operating firms and therefore rent dissipation. Most importantly, it simultaneously alters the distribution of prices in the market and, through search, the distribution of buyers across prices. This complex allocative effect can potentially increase or decrease the inefficiency of monopolistic

pricing, depending on preferences and market structure. In markets for goods which consumers highly value or where search is relatively inexpensive, welfare will tend to rise with the inflation rate, as most buyers take advantage of the increased price dispersion. When these conditions fail, inflation can significantly decrease welfare. Whether welfare-promoting or reducing, these effects are limited in equilibrium by search and the exit of firms, which act as restricting forces on the dispersion of prices.

Inflation was defined here in the traditional sense, as a smooth and anticipated growth rate in aggregate prices. One might expect that if increased inflation is defined as (or accompanied by) an increase in the variance of nominal aggregate prices rather than in their trend, it will interfere with the informational content of the price system and have more severe welfare consequences. Benabou and Gertner [1988] examine this issue, and show that when the possibility of acquiring information through search is recognized, similarly guarded conclusions emerge.

Clearly, this paper has only focused on some of the effects of inflation. Some other ones could be incorporated into the model, such as a Diamond-type thin-market externality, causing the distribution of search costs to shift up as the number of firms fall; But undoubtedly many other of the potential costs identified by Fischer and Modigliani [1978] (e.g. shoe-leather effects, managerial resources devoted to insure against or speculate on inflation, etc.) lie outside the scope of the paper. Its aim, however, was only to formally model and assess a certain conventional wisdom about inflation's impact on welfare, operating specifically through the price system and several types of transactions costs. In that respect, the lesson may be that simple arguments about the "distortions" inflicted by inflation to the price system should probably be received (and put forward) with caution.

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APPENDIX

Proof of Proposition 2:

Let $K \equiv \bar{\gamma} - \max(\underline{\gamma}, \overline{V}(S,s))$, and $\Delta(p) \equiv z(p)(V(p) + K)$, for all p. On $[\underline{r}, \bar{r})$, D(p) is proportional to $\Delta(p)$ and $\Pi(p)$ to $\Pi_\Delta(P) \equiv (p-c)\Delta(p)$, which will now be shown to be strictly quasiconcave on its support $[c,\bar{r}]$ $[c,M]$. There are two cases to consider.

Case 1: When $K > 0$, or $\bar{r} < M$, we show the sronger property that $\Delta(p)$ has increasing elasicity $\alpha_\Delta(p)$. Proposition 1 will then yield strict quasiconcavity of $\Pi_\Delta(p)$. On $(0,\bar{r})$ $\Delta(p)$ has elasticity:

$$\text{(A1)} \qquad \alpha_\Delta(p) = \alpha_z(p) + \frac{pz(p)}{V(p) + K} > \alpha_\Delta(p)$$

and:

$$\text{(A2)} \qquad \alpha_\Delta'(p) = \alpha_z'(p) + \frac{z(p)(1-\alpha_z(p))(V(p) + K)) + p.z(p)^2}{(V(p) + K)^2}$$

where we used the fact that:

$$\text{(A3)} \qquad \frac{d(p.z(p))}{dp} = z(p)(1- \alpha_z(p)).$$

But this same equality yields:

$$p.z(p) - \bar{r}.z(\bar{r}) = - \int_p^{\bar{r}} z(u)(1- \alpha_z(u))du$$

$$= (V(\bar{r}) + K)(1 - \alpha_z(\bar{r})) - (V(p) + K)(1 - \alpha_z(p))$$

$$+ \int_p^{\bar{r}} (V(u) + K).\alpha_z'(u)du$$

The numerator of the second term in (A2) thus becomes:

$$\text{(A4)} \qquad z(p)[\bar{r}.z(\bar{r}) + \int_p^{\bar{r}} (V(u) + K).\alpha_z'(u)du - (V(\bar{r}) + K)(\alpha_z(\bar{r}) - 1)$$

By definition of $\bar{r}$ and K, $\bar{r} < M$ implies $V(\bar{r}) + K = 0$, so the last term cancels out, while the remaining two are positive; hence the result.

Case 2: The case where $K \geq 0$, or $\bar{r} = M$ is more difficult, because the last term in (A4) is generally negative. Consider marginal profits on $[0,M]$:

$$(A5) \qquad \Pi_\Delta'(p) = \Delta'(p)[p(1 - 1/\alpha_\Delta(p)) - c\,] \equiv \Delta'(p)[h(p) - c]$$

where $\alpha_\Delta(p) > \alpha_z(p)$ was defined in (A2). By Assumption 1, α_z is non-decreasing, so there exists an $m_z \in [0,M]$ such that $\alpha_z(p) \leq 1$ on $(0,m_z]$ and $\alpha_\Delta(p) \geq 1$ on $[m_z,M)$; one of these intervals may be empty, but when $M = +\infty$, $\alpha_z(+\infty) > 1$ implies $m_z < +\infty$.

On $(0,m_z)$, $\alpha_z(p) \leq 1$ and (A3) imply that the function $p.z(p)$ is non-decreasing. Therefore, by (A1) and (A2), α_Δ is strictly increasing on $[0,m_z]$, so there exists a unique $m_\Delta \in [0,m_z]$, such that $\alpha_\Delta(p) \leq 1$ on $(0,m_\Delta]$ and $\alpha_\Delta(p) > 1$ on $[m_\Delta,M)$.

On $[0,m_\Delta]$, $\alpha_\Delta(p) \leq 1$, so $h(p) \leq 0$ by (A5), or $\Pi_\Delta'(p) > 0$. We shall now examine the variations of $h(p)$ on $[m_\Delta,M]$.

i) On $(m_\Delta,m_z]$, $\alpha_\Delta(p)$ is increasing and and $1 - 1/\alpha_\Delta(p) > 0$, so $h(p)$ is increasing.

ii) On $(m_z,M]$ (when it is not empty) $\alpha_\Delta(p)$ need not be monotonic but we shall prove that $h(p)$ is. Indeed:

$$\alpha_\Delta(p)^2.h'(p) = \alpha_\Delta(p)^2 - \alpha_\Delta(p) + p.\alpha_\Delta'(p).$$

But multiplying (A2) by p and substituting in (A1) yields:

$$p.\alpha_\Delta'(p) = p.\alpha_z'(p) + (1 - \alpha_z(p))(\alpha_\Delta(p) - \alpha_z(p)) + (\alpha_\Delta(p) - \alpha_z(p))2$$

$$\geq (\alpha_\Delta(p) - \alpha_z(p))(1 + \alpha_\Delta(p) - 2\alpha_z(p))$$

so that (omitting the dependence on p):

$$\alpha_\Delta{}^2.h' \geq \alpha_\Delta{}^2 - \alpha_\Delta + (\alpha_\Delta - \alpha_z)(1 + \alpha_\Delta - 2\alpha_z)$$

$$= \alpha_z(\alpha_z - 1) + (\alpha_\Delta - \alpha_z)(\alpha_\Delta + \alpha_z - 1 + 1 + \alpha_\Delta - 2\alpha_z)$$

$$= \alpha_z(\alpha_z - 1) + (\alpha_\Delta - \alpha_z)(2\alpha_\Delta - \alpha_z) > 0$$

since $\alpha_\Delta > \alpha_z > 1$. Thus $h(p)$ increases on $(m_\Delta, M]$ an is non-positive on $[0, m_\Delta]$. As a result, there exists a unique $p^* \in (m_\Delta, M]$, such that $h(p) < c$, i.e. $\Pi_\Delta'(p) > 0$ on $(0, p^*)$ and $h(p) > c$, i.e. $\Pi_\Delta'(p) < 0$ on $(p^*, M]$. Moreover, $\Pi_\Delta'(p^*) = 0$ if $p^* < M$, which always is the case when $M = +\infty$, because $\alpha_\Delta(+\infty) \geq \alpha_z(+\infty) > 1$ implies $h(+\infty) = +\infty$. Q.E.D.

Proof of Theorem 2:

Given ν, an equilibrium (S,s) is a fixed point of firms' best reply function. With endogenous entry, however, it is a solution to (27)-(28), which corresponds to a fixed point of a somewhat different mapping, with a less straightforward economic interpretation.

1. Preferences with bounded support:

Assume first that $M < +\infty$, so that the non-empty set:

(A6) $$K \equiv \{(S,s) \in R^2 \mid c \leq s \leq p^m,\ s \leq S \leq M\}.$$

is not only convex but also compact. For all (S,s) in K, define as before the average surplus $\overline{V}(S,s)$ by:

(A7) $$\overline{V}(S,s) \equiv \int_s^S \frac{V(u)}{u} \cdot \frac{du}{Ln(S/s)}$$

for $s < S$, and by $\overline{V}(s,s) = V(s)$ (l'Hopital's rule). Clearly:

(A8) $$V(S) \leq \overline{V}(S,s) \leq V(s)$$

with strict inequality unless $S = s$. Define now, for all (S,s) in K and any $p \in [c,M]$:

$$\text{(A9)} \qquad \Pi_{(S,s)}(p) \equiv \pi(p).\max\{\, V(u) + \tilde{\gamma} - \overline{V}(S,s),\ 0 \,\}$$

which is, up to a constant, the profit function of a firm in an (S,s) equilibrium. From Proposition 1, we know that $\Pi_{(S,s)}(p)$ is strictly quasiconcave (and clearly continuous) on its support, which is $[c,\tilde{r}]$. Note that if $s < M$, $\Pi_{(S,s)}(p) > 0$ for p just above s, while if $s = M$, then $S = s = M$ so $\Pi_{(S,s)}(p) = \tilde{\gamma}\pi(p) > 0$ for p just above c. Thus $\Pi_{(S,s)}$ never has trivial support ($\tilde{r} > c$).

Next, define for all (S,s) in K:

$$\text{(A10)} \qquad J(S,s) \equiv \int_s^S \Pi_{(S,s)}(u)\frac{u^{\delta-1}}{(S^\delta - s^\delta)/\delta + g\beta S^\delta/h}\,du.$$

J(S,s) is a weighted average of a firm's discounted profits in an (S,s) equilibrium, i.e. of $\rho W(S,s)$, given by (12), and of h; with entry, $\rho W(S,s) = h$, so $J(S,s) = \rho W(S,s) = h$.

Denoting by $p^*_{(S,s)} \leq p^m$ the unique maximum of the function $\Pi_{(S,s)}$, and by $\Pi^*_{(S,s)}$ its maximal value, we have:

$$\text{(A11)} \qquad 0 \leq J(S,s) < \Pi^*_{(S,s)} \cdot \frac{(S^\delta - s^\delta)/\delta}{(S^\delta - s^\delta)/\delta + g\beta S^\delta/h} < \frac{\Pi^*_{(S,s)}}{1 + g\beta/h}$$

with the first inequality being strict unless $S = s$ (since $S < +\infty$). The strict quasiconcavity of $\Pi_{(S,s)}(p)$ and (A11) imply:

$$\text{(A12)} \quad \begin{cases} \forall\ (S,s) \in K, \exists\ !\ (s',S') \in [c,\ p^*_{(S,s)}) \times (p^*_{(S,s)},\ M] \subset K \text{ such that} \\ \Pi_{(S,s)}(s') = J(S,s); \\ \Pi_{(S,s)}(S') \geq (1 + \rho\beta/h)J(S,s); \\ [\Pi_{(S,s)}(S') - (1 + \rho\beta/h)J(S,s)](M - S) = 0; \end{cases}$$

Moreover, note that: $s' > c$ unless $J(S,s) = 0$, i.e. unless $S = s$. We shall

denote the solution to (A12), given (S,s), by $(S',s') = \Psi(S,s)$.

The functions appearing in (A12) are clearly continuous in (S,s;S',s'). Together with the uniqueness of the solution (S',s'), this implies that Ψ is continuous in (S,s) on K. Indeed, if $(S_n,s_n)_{n\in N}$ converges to $(S,s) \in K$, then the corresponding sequence $(S'_n,s'_n)_{n\in N}$ is in the compact set K, so it must have at least one accumulation point (S",s"). But writing down (A12) for $(S_n,s_n;S'_n,s'_n)$ and taking limits implies that (S,s;S",s") verifies (A12). The uniqueness of the solution, given (S,s), implies that any such accumulation point (S",s") must equal (S',s'); thus $(S'_n,s'_n)_{n\in N}$ converges to (S',s').

The function Ψ, which maps the convex, compact set K into itself, must have a fixed point $(S^*,s^*) = \Psi(S^*,s^*) = (S^{*\prime},s^{*\prime})$. Moreover, by (A12), $s^{*\prime} < p^*_{(S,s)} < S^{*\prime}$, so: (i) $s^* < p^m$; (ii) $s^* < S^*$, hence $J(S^*,s^*) > 0$ and $s^* > c$. This ensures that $x \in (0,+\infty)$. Similarly, if $z(M) = 0$, then necessarily $S < M$. This concludes the proof in this case.

2.Preferences with unbounded support:

Assume now that z(p) and V(p) have support $(0,+\infty)$. For each finite $M > p^m$, consider the truncated demand function $z_M(p)$ (or surplus $V_M(p)$) which coincides with z(p) (or V(p)) on (0,M], and is zero afterwards. Since z_M still satisfies Assumption 1, the above result guarantees the existence of an equilibrium (S^*_M,s^*_M) for these modified preferences. The equilibrium (S^*,s^*) for the infinite-support-problem will be constructed as a limit of (S^*_M,s^*_M) for a sequence of values of M going to $+\infty$. In order to do so, it must be proved that S^*_M remains bounded even as M tends to $+\infty$. We shall start by proving:

Lemma 1: There exists $B > p^m$, such that for all $s \in [c,p^m]$, and all $S > B$:

$$\text{(A13)} \qquad \frac{J(S,s)}{\Pi_{(S,s)}(S)} > \frac{1}{1 + \rho\beta/h}$$

Moreover, the same inequality holds, when $B < S < M$, for the functions $\Pi^M_{(S,s)}(p)$ and $J^M(S,s)$ associated by (A7)-(A9)-(A10) to any finite truncation of preferences at $M > B$.

This result will prove that for all $M > B$, $S^*_M \leq B$, or else the second condition of (A12), for the corresponding functions $\Pi^M_{(S,s)}(p)$ and $J^M(S,s)$, would not be verified with $S^{*\prime}_M = S^*_M$. Note that, since $V(p)$ is decreasing and $s > 0$:

$$\frac{J(S,s)}{\Pi_{(S,s)}(S)} = \int_s^S \frac{\pi(u)(V(u) + \bar{\gamma} - \bar{V}(S,s)}{\pi(S)(V(p) + \bar{\gamma} - \bar{V}(S,s)} \cdot \frac{\delta u^{\delta-1}}{S^\delta(1+\rho\beta/h) - s^\delta}\, du$$

$$> \int_s^S \frac{\pi(u)}{\pi(S)} \cdot \frac{\delta u^{\delta-1}\, du}{S^\delta(1+\rho\beta/h)} .$$

The same holds true in any truncated problem, so that it suffices to prove:

Lemma 2: There exists $B > p^m$, such that for all $s \in [c,p^m]$ and all $S > B$:

$$\text{(A14)} \qquad \int_s^S \frac{\pi(u)}{\pi(S)} \frac{\delta u^{\delta-1}}{S^\delta}\, du > 1$$

Moreover, the same inequality also holds, for $B < S < M$, for the function $\pi^M(p) = (p-c)z^M(p)$ associated to any truncation of preferences at $M > B$.

Proof: For $M > S > B$, the left-hand-side of (A14) remains unchanged when preferences are truncated at M, so that attention can be confined to the untruncated case. Under Assumption 1, there exists $A > p^m$ and $\alpha > 1$ such that, for all $p \geq A$: $z'(p)/z(p) > -\alpha/p$. Integrating over $[u,S]$ for $u < S$:

$$\text{(A15)} \qquad \frac{z(S)}{z(u)} < \left[\frac{S}{u}\right]^{-\alpha}$$

The left hand side of (A14) is therefore bounded from below by:

$$(A16) \qquad \int_A^S \frac{(u-c)}{(S-c)} \cdot \frac{u^{\delta-\alpha+1}}{S^{\delta-\alpha}}\, du = \frac{\delta}{S^{\delta-\alpha}(S-c)}\left[\frac{S^{\delta-\alpha+1} - A^{\delta-\alpha+1}}{\delta - \alpha + 1} - \frac{S^{\delta-\alpha} - A^{\delta-\alpha}}{\delta - \alpha}\right]$$

where we have assumed, for now, that $\delta - \alpha \notin \{0,-1\}$.

Case 1: $\delta - \alpha + 1 > 0$; as S tends to $+\infty$, the terms in $S^{\delta-\alpha+1}$ dominate the above expression, which therefore tends to $\delta/(\delta - \alpha + 1) > 1$, hence the result.

Case 2: $\delta - \alpha + 1 < 0$: the term in brakets is always positive, and in the limit both $S^{\delta-\alpha+1}$ and $S^{\delta-\alpha}$ tend to zero; since the first term is equivalent to $\delta/S^{\delta-\alpha+1}$, the above expression tends to $+\infty$ with S, hence the result.

Case 3: $\alpha - \delta + 1 = 0$; the right-hand side of (A16) must then be replaced by $[\delta S/(S-c)][\mathrm{Ln}(S/A) + c.\,(1/A - 1/S)]$, which also tends to $+\infty$ with S.

Case 4: $\alpha - \delta = 0$; the right-hand side of (A16) must then be replaced by $\delta[(S-A)/(S-c) - c.\mathrm{Ln}(S/A)/(S-c)]$, which also tends to $+\infty$ with S.

This concludes the proof of Lemma 2, in all cases. Consider now a sequence of truncations at M = n, for $n \in N$, $n > B$, with corresponding equilibria (S_n^*, s_n^*). By definition, $(S_n^*, s_n^*, S_n^*, s_n^*)$ satisfy (A12), for the functions $\Pi^n_{(S,s)}(p)$ and $J^n(S,s)$. By Lemma 1, the sequence $(S_n^*, s_n^*)_{n\in N}$ remains in the compact set $[c,p^m]x[p^m,B]$, so there exists a subsequence which converges to some limit (S^*,s^*); taking limits in (A12) for that subsequence easily yields the result that $(S^*,s^*;S^*,s^*)$ satisfies (A12) for the untruncated functions z(p) and V(p), so that (S^*,s^*) is an equilibrium for the original preferences with unbounded support. Q.E.D.

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